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# ON D-ECCENTRICITY MATRIX AND D-ECCENTRICITY ENERGY

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**Abstract-** For two vertices u and v of a graph G, d(u,v) is the length of the shortest path between u and V. D. Reddy Babu and L.N. Varma introduced the concept of D-distance.
D-distance considers the degree of all vertices present in a path while defining its length.
In this paper, the D- eccentricity matrix is defined and D- eccentricity energy of some class of graphs are found**.**

**Keywords**— Distance, D-distance, eccentricity matrix,sepectrum, energy.

## I. INTRODUCTION

The theory of Linear Algebra, in particular theory of matrices is a powerful tool to study the spectral properties of the graph spectra and in turn matrix properties of the graph can be recognized from the spectrum of its matrix.
The concept of energy of graph was put forward by I.Gutman in 1978 .
By a graph G, we mean non trivial, finite and undirected graph whithout multiple edges and loops.
In graph G, the usual distance d (u, v) is the length of the minimum path connecting the vertices u and v of G.
The D-distance $d^D(u,v)$ between two vertices of a connected graph G is defined as
$d^D(u,v) = \min\{d(u,v) + \deg(u) + \deg(v) + \sum \deg(w)\}$ where sum runs over all the intermediate vertices w in the path and minimum is taken over all u-v paths in G[2].
In this article, inspired by the definition of eccentricity matrix $\varepsilon(G)$ and eccentricity energy of some class of graphs[1,2], we define D-eccentricity matrix $D\varepsilon(G)$ and find D-eccentricity energy of some class of graphs.

## II D- eccentricity MATRIX and D-eccentricity ENERGY

**Definition 2.1:**

The D-eccentricity of any vertex v, $e^D(v)$ is defined as the maximum D-distance from v to any other vertex that is, $e^D(v) = \max\{d^D(u,v): u \in V(G)\}$[2]

**Definition 2.1.1:**

If $\beta_1, \beta_2, \beta_3 \dots \dots .. \beta_n$ are the eigenvalues of D-distance eccentricity matrix of the corresponding graph G then D-eccentricity energy is given by $E^*(G) = \sum_{i=1}^{n} \beta_i$ .

The elements of D-eccentricity matrix is defined by

$$D\varepsilon(G) = \begin{cases} dij^D & if\ dij^D = \min\{e^D(u_i), e^D(v_j)\} \\ 0 & if\ \ dij^D < \min\{e^D(u_i), e^D(v_j)\} \end{cases}$$

The $D\varepsilon$-spectram of a graph consists of $D\varepsilon$ eigenvalues of its D-eccentricity matrix.

## III. RESULTS

**Definition 3.1:** A graph in which each pair of vertices is connected by an edge is a complete graph.

**Theorem 3.1.1:** The D-eccentricity energy of a complete graph $K_n$ with n vertices and m edges for n > 2 is $(2n-2)(2n-1)$.

**Proof:**

Let $K_n$ be a complete graph with n vertices $\{v_1\, v_2\, \ldots .\, v_n\}$ with m edges that is each vertex is adjacent to every other vertex. Therefore, D- distance from one vertex to any other vertex is given by $d_{ij}^D = d_i + d_j + 1 = \deg(u) + \deg(v) + d(u,v)$.

D-eccentricty of each vertex is also $d_{ij}^D = d_i + d_j + 1$.

For complete graph $K_n$, D-eccentricity matrix is given by

$$D\varepsilon(K_n) = \begin{bmatrix} 0 & 2n-1 & 2n-1 & 2n-1 & \ldots\ldots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 0 & 2n-1 & 2n-1 & \ldots\ldots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 0 & 2n-1 & \ldots\ldots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 0 & \ldots\ldots & 2n-1 & 2n-1 & 2n-1 & 2n-1 \\ \vdots & \vdots & \vdots & \vdots & \ldots\ldots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ldots\ldots & \vdots & \vdots & \vdots & \vdots \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \ldots\ldots & 2n-1 & 0 & 2n-1 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \ldots\ldots & 2n-1 & 2n-1 & 0 & 2n-1 \\ 2n-1 & 2n-1 & 2n-1 & 2n-1 & \ldots\ldots & 2n-1 & 2n-1 & 2n-1 & 0 \end{bmatrix}$$

The characteristic equation is $[\beta + (2n-1)]^{n-1}[\beta - (n-1)(2n-1)] \quad = 0$

The D-eccentricity eigenvalues are

$\beta = (2n-1)(n-1)$ [one time], $\beta = -(2n-1)$ [$(n-1)$times]

D-eccentricity energy of $K_n$ is $E^*(G) = (n-1)|-(2n-1)| + (2n-1)(n-1)$

$= (2n-2)(2n-1)$

**Corollary 3.1.2:**

D-eccentricity energy of $K_n = (2n-1)$ times eccentricity energy of $K_n$ obtained by usual distance.

**Proof:**

The eccentricity energy of a complete graph $K_n$ with n vertices and m edges for n > 2 is

$(2n-2)$ [3]

Hence, the result.

**Example 3.1.3**

A complete graph with 5 vertices (n = 5)

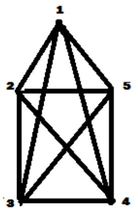


Fig 1

The D-eccentricity matrix of the complete graph in fig 1 is

$$D\varepsilon(K_5) = \begin{bmatrix} 0 & 9 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 9 \\ 9 & 9 & 0 & 9 & 9 \\ 9 & 9 & 9 & 0 & 9 \\ 9 & 9 & 9 & 9 & 0 \end{bmatrix}$$

Characteristic equation of $K_5\ is\ [\beta + 9]^4[\beta - 36]^1 = 0$

$\beta = 36\ [one\ time], \beta = -9\ [\ four\ times]$

D-eccentricity energy of $K_5$ is $E^*(G) = 4|-9| + 36 = 72$

The eccentricity energy of a complete graph $K_5$ with 5 vertices for n > 2, is 8

Therefore, D-eccentricity energy of $K_5$ is 9(8) = 72

**Definition 3.2:**

A graph consisting of two rows of a paired vertices in which all the vertices except the paid ones are joined by an edge denoted by $CP_k$ where k=2n, for all $n \geq 2$ is a cocktail party graph.

**Theorem 3.2.1:**

The D-eccentricity energy of a cocktail party graph $CP_k$ where k=2n, for all $n \geq 2$ is

$4n\ (3n - 2)$.

**Proof:**

Let $CP_k$ where k=2n, for all $n \geq 2$ be a cocktail party graph with vertex set $V = \cup_{i=1}^{n}\{u_i, v_i\}$.

Each vertex $v_i$ is adjacent to every (n-1) vertices in the vertex set $\{u_i\}$.

The D-distance are $d^D(u_i, u_i) = 0$ and $d^D(v_i, v_i) = 0$

$$d^D(v_i, u_j) = \begin{cases} 1 + 2(2n - 1)\ if\ i\ \neq j \\ 2 + 3(2n - 2)\ if\ i = j \end{cases}$$

Hence, by the definition of D-eccentricity matrix we have

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$D\varepsilon(CP_k) =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & \dots\dots & 2+3(2n-1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots\dots & 0 & 2+3(2n-1) & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots\dots & 0 & 0 & 2+3(2n-1) & 0 \\ 0 & 0 & 0 & 0 & \dots\dots & 0 & 0 & 0 & 2+3(2n-1) \\ \vdots & \vdots & \vdots & \vdots & \dots\dots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \dots\dots & \vdots & \vdots & \vdots & \vdots \\ 2+3(2n-1) & 0 & 0 & 0 & \dots\dots & 0 & 0 & 0 & 0 \\ 0 & 2+3(2n-1) & 0 & 0 & \dots\dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 2+3(2n-1) & 0 & \dots\dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+3(2n-1) & \dots\dots & 0 & 0 & 0 & 0 \end{bmatrix}$$

The characteristic equation is $\{\beta + [2+3(2n-2)]\}^n\{\beta - [2+3(2n-2)]\}^n = 0$

The D-eccentricity eigenvalues are

$\beta = [2+3(2n-2)]\ [n\ times], \beta = -[2+3(2n-2)]\ [n\ times]$

D-eccentricity energy of $CP_k$ is $E^*(G) = (n)|[2+3(2n-2)]| + (n)|-[2+3(2n-2)]|$

$$= 4n(3n-2)$$

**Corollary 3.2.2:**

D-eccentricity energy of $CP_k = (3n-2)$ times eccentricity energy of $CP_k$ obtained by usual distance.

**Proof:**

The eccentricity energy of a cocktail party graph $CP_k$ where k=2n, for all $n \geq 2$ is

$4n$ [3]

Hence, the result.

**Example 3.2.3:**

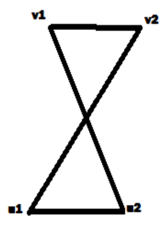


Fig 2

The D-eccentricity matrix of cocktail party graph in fig 2 is

$$D\varepsilon(CP_4) = \begin{bmatrix} 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \\ 8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \end{bmatrix}$$

 

Characteristic equation of $CP_4$ *is* $[\beta+8]^2[\beta-8]^2=0$

$\beta=8\,[two\ times], \beta=-8\,[two\ times]$

D-eccentricity energy of $CP_4$ is $E^*(G)=2|-8|+2|8|=32$

The eccentricity energy of a cocktail party graph $CP_4$ with n=2, is 8

Therefore, D-eccentricity energy of $CP_4$ is 4(8) = 32

**Definition 3.3.1:**

Crown graph on 2n vertices is an undirected graph with two set of vertices $u_i$ *and* $v_j$ and with an edge $u_i$ *to* $v_j$ where $i \neq j$.

**Theorem 3.3.2:**

The D-eccentricity energy of a Crown graph $S_k$ where k = 2n for $n > 2$ is $n(6n)+2n(n-1)$.

**Proof:**

Suppose $S_k$ is a Crown graph with k vertices. Then the vertex set of $S_k$ is partitioned into two subsets $V_1$ *and* $V_2$ such that $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

Let $V_1=\{v_1, v_2 \ldots\ldots v_n\}$ and $V_2=\{u_1, u_2 \ldots\ldots u_n\}$ be the two subsets of $V(S_k)$ and all vertices of $V_1$ are connected to each vertex of $V_2$ except paired ones.

The D-distance are

$$d^D(u_i,u_j)=\begin{cases}3(n-1)+2 & if\ i \neq j\\ 0 & if\ i=j\end{cases}$$

$$d^D(v_i,v_j)=\begin{cases}3(n-1)+2 & if\ i \neq j\\ 0 & if\ i=j\end{cases}$$

$$d^D(v_i,u_j)=\begin{cases}2(n-1)+1 & if\ i \neq j\\ 3+4(n-1) & if\ i=j\end{cases}$$

Hence, by the definition of D-eccentricity matrix we have

$D\varepsilon(CP_k)=$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & \ldots\ldots & 3+4(n-1) & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \ldots\ldots & 0 & 3+4(n-1) & 0 & 0\\
0 & 0 & 0 & 0 & \ldots\ldots & 0 & 0 & 3+4(n-1) & 0\\
0 & 0 & 0 & 0 & \ldots\ldots & 0 & 0 & 0 & 3+4(n-1)\\
\vdots & \vdots & \vdots & \vdots & \ldots\ldots & \vdots & \vdots & \vdots & \vdots\\
\vdots & \vdots & \vdots & \vdots & \ldots\ldots & \vdots & \vdots & \vdots & \vdots\\
3+4(n-1) & 0 & 0 & 0 & \ldots\ldots & 0 & 0 & 0 & 0\\
0 & 3+4(n-1) & 0 & 0 & \ldots\ldots & 0 & 0 & 0 & 0\\
0 & 0 & 3+4(n-1) & 0 & \ldots\ldots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 3+4(n-1) & \ldots\ldots & 0 & 0 & 0 & 0
\end{bmatrix}$$

The characteristic equation is $\{\beta+[3+4(n-1)]\}^n\{\beta-[3+4(n-1)]\}^n=0$

The D-eccentricity eigenvalues are

$\beta = [3 + 4(n - 1)]\ [n\ times], \beta = -[3 + 4(n - 1)]\ [n\ times]$

D-eccentricity energy of $CP_k$ is $E^*(G) = (n)|[3 + 4(n - 1)]| + (n)|-[3 + 4(n - 1)]|$

$$= n(6n) + 2n(n - 1)$$

**Corollary 3.3.3:**

D-eccentricity energy of $S_k = n$ times eccentricity energy of $S_k + 2n(n - 1)$ obtained by usual distance.

**Proof:**

The eccentricity energy of a Crown graph $S_k$ where k=2n, for all $n > 2$ is

$6n$ [3]

Hence, the result.

**Example 3.8:**

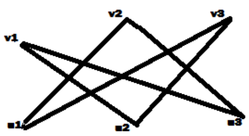


Fig 3

The D-eccentricity matrix of cocktail party graph in fig 3 is

$$D\varepsilon(S_6) = \begin{bmatrix} 0 & 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 \\ 11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 & 0 \end{bmatrix}$$

Characteristic equation of $S_6\ is\ [\beta + 11]^3[\beta - 11]^3 = 0$

$\beta = 11\ [three\ times], \beta = -11[\ three\ times]$

D-eccentricity energy of $S_6$ is $E^*(G) = 3|-11| + 3|11| = 66$

The eccentricity energy of a cocktail party graph $S_6$ with n=3, is 18

Therefore, D-eccentricity energy of $S_6$ is 3(18)+2(3)(3-1) = 66

## IV. CONCLUSION

In this article we have found the D-eccentricity energy of complete graph, cocktail party graph and crown graph by using D-eccentricity matrix.

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