

THEORETICAL MODELLING OF STRATIFIED CONDENSATION OF TWO-PHASE CO-CURRENT FLOW INCLUDING PRESSURE DROP IN INCLINED TUBES

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Abstract— The investigation of stratified condensation of two-phase flow inside inclined tube is done theoretically in order to simplify the study of condensation heat transfer coefficient. The effect of pressure gradient, interfacial shear and stratified angle is studied in conjunction with the effect of inclination angle. A simplified numerical approach is mentioned in order to achieve the variation of heat transfer coefficient in axial direction of inclined tube. Various governing differential equations are solved simultaneously considering the effect of pressure drop to obtain the heat transfer rate. The solution of these governing equations is achieved by using explicit discretization scheme. The results thus obtained are compared with the reported theoretical and experimental data and are found to be in good agreement.

Keywords— Stratified flow, inclination angle, pressure drop, heat transfer coefficient

I. INTRODUCTION

The rising demand of condensers in various refrigeration industries attracted many researchers towards the in-depth study of condensation phenomenon. Various flow patterns developing inside condenser tubes were analysed with their respective effects. [1] and [3] gives a short review of various works done on condensation analysis. Chen and Kocamustafaogullari [2] (1986) designed a theoretical model in order to obtain the variation of heat transfer coefficient across the tube length. Regression analysis was used for developing the correlation. Hamid Saffari et al. [4] developed theoretical model for condensation inside inclined tube. C. E. Rufer et al. [5] analyzed stratified flow considering effect of different tube inclination angles. Experimental analysis was conducted by Goodykoontz and Dorsch [6] on steam condensation taking place inside a tube of inner diameter 5/8 inch. with 8 foot length. Complete condensation was observed across the test section. Their conclusion was that the heat transfer coefficients were highly depended on the vapor flow rate.

II. MATHAMETICAL MODEL

The model used for present study is as depicted in figure given below (Figure 1). The model consists of a tube having a circular cross-section which is kept inclined to an angle ' θ ' with horizontal axis. The z-axis is placed along the axial length of the tube i.e. z-axis gives variation of flow along the length of the tube. Whereas x-axis represents tangential direction of flow and y-axis gives radial direction of flow. Saturated vapor is made to enter the tube from inlet kept at $z=0$. The saturated vapor is made to flow in downward direction along the axis of the tube. The wall of tube is kept at a temperature lower than that of inlet saturation temperature of vapor. Due to this temperature difference, the saturated vapor flowing inside the tube gradually gets condensed forming a thin liquid film on the tube wall. Thus, condensate flow is observed throughout the length of the tube.

The condensate film thus formed is observed to flow along the x-axis covering the peripheral region of the tube with a peripheral angle ' ϕ ' as shown in Figure 1.b. This film starts moving in downward direction due to gravitational effects. Further the condensate film was assumed to get accumulate in bottom region of tube, resulting in low heat transfer rate in this region due to high thermal resistance.

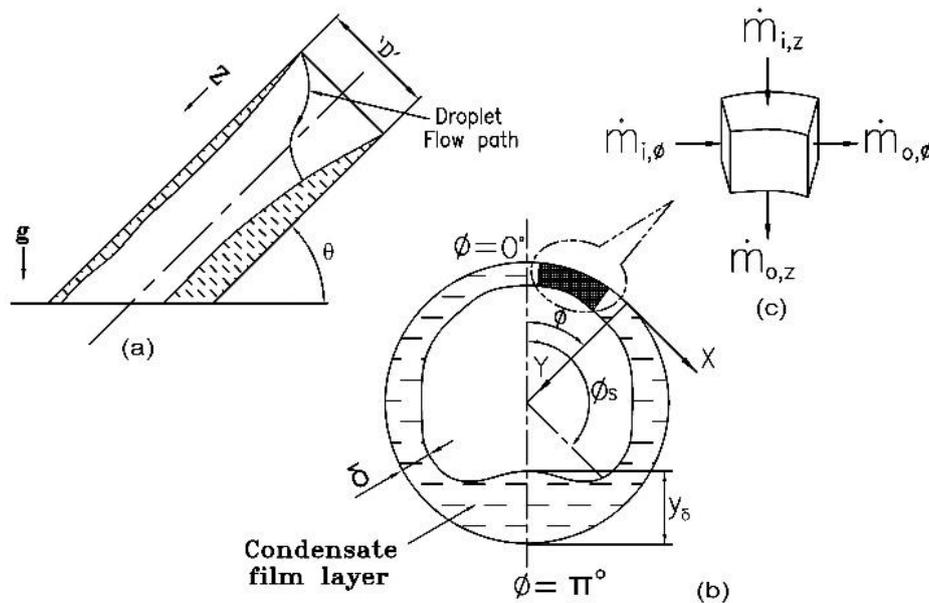


Figure 1 Coordinate system and physical model for condensate film flow and accumulated layer.

A. Assumptions

Several more assumptions were made to simplify the current study. Which are listed below: -

1. The flow of condensed liquid film is assumed to be Stratified, laminar and steady.
2. The working fluid is considered to be pure.
3. Inner wall of circular tube is assumed to be smooth.
4. The tube wall temperature is maintained at some constant temperature (T_w) which is lower than saturated temperature (T_{sat}) of incoming vapor. The temperature of condensed film at the vapor-liquid interface is assumed to be equal to that of saturated temperature of incoming vapor and is therefore kept constant.
5. The properties of fluid are evaluated at bulk mean temperature (reference temperature) given by $0.5*(T_w - T_{sat})$ and are assumed to be constant, except the density term in the momentum equation.
6. The surface tension is neglected as the condensate film thickness is assumed to be very small when compared with the tube diameter. Hence convection heat transfer in the condensate film is negligible whereas conduction heat transfer from the condensate film surface to the tube wall is considerable.
7. The change of tangential and axial velocities in the peripheral and axial directions is assumed to be negligibly small as compared to their change across y-direction (radial direction).

The study on condensation inside a tube can be divided in two parts; one part will be related to analysis of falling condensate film flow and the other part will represent the impact of accumulated condensate layer on heat transfer coefficient of condenser.

B. Mathematical Analysis

As stated above, the condensation analysis will be divided in two parts; the angle at which development of accumulated condensate layer will start will be taken as limit to separate both these parts. The above analysis will be started by considering basic conservation equations that are conservation of mass, conservation of energy and conservation of momentum.

C. Analysis of flowing condensate layer, ($0 \leq \phi \leq \phi_s$)

Considering 'u', 'v' and 'w' be the components of velocity in 'x', 'y' and 'z' direction respectively of flow flowing through a tube of diameter 'D' and tube length 'L' inclined at an angle 'theta', the governing equations are given as

Mass conservation equation:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

Conservation of momentum in x-direction:

$$\mu_l \frac{\partial^2 u}{\partial y^2} + (\rho_l - \rho_g)g \sin \phi \cos \theta = 0 \quad (2)$$

Conservation of momentum in z-direction:

$$\mu_l \frac{\partial^2 w}{\partial y^2} + (\rho_l - \rho_g)g \sin \theta - \frac{dP}{dz} = 0 \quad (3)$$

Further the energy conservation equation can be given as:

$$\frac{\partial^2 T}{\partial y^2} = 0 \quad (4)$$

Let the film thickness be denoted as ‘ δ ’ which is assumed to be much smaller than tube diameter.

The boundary conditions for solving all the above-mentioned, equations are as follows:

$$\text{At } y=0; u=w=0 \text{ and } T=T_w, \quad (5)$$

$$\text{At } y=\delta; \frac{\partial u}{\partial y} = 0, \text{ and } \mu \left(\frac{\partial u}{\partial y} \right) = \tau_i \quad (6)$$

Also;

$$T=T_{sat}, \text{ and } \dot{m}_i'' h_{lg} = -k_l \left(\frac{\partial T}{\partial y} \right) \quad (7)$$

Where \dot{m}_i'' is the condensate mass flux and is given by

$$\dot{m}_i'' = -\rho \left(u \frac{\partial \delta}{\partial x} + w \frac{\partial \delta}{\partial z} \right) \text{ at } y = \delta \quad (8)$$

Using the boundary conditions to integrate equation (2) and (3) across the thickness of condensate film, the values of ‘ u ’ and ‘ v ’ respectively are obtained as

$$u = \frac{(\rho_l - \rho_g)g \sin \phi \cos \theta}{\mu_l} \delta^2 \left[\frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^2 \right] \quad (9)$$

$$w = \frac{(\rho_l - \rho_g)g \sin \theta}{\mu_l} \delta^2 \left[\frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^2 \right] + \left(\frac{dP}{dz} \right) \left(\frac{1}{\mu_l} \right) \left[\frac{1}{2} \left(\frac{y}{\delta} \right)^2 - \frac{y}{\delta} \right] + \frac{y}{\mu_l} \tau_i \quad (10)$$

A linear variation in temperature was observed after integration of energy equation, equation (4)

$$T = T_w + (T_{sat} - T_w) y / \delta \quad (11)$$

Further to obtain local film thickness as a function of stratified angle and length of tube, the equations (9) and (10) are substituted in equation (1) by simultaneously integrating the continuity equation in the limits of ‘0’ to ‘ δ ’ with respect to ‘ y ’

$$\frac{\partial}{\partial x} \int_0^\delta u dy + \frac{\partial}{\partial z} \int_0^\delta w dy = (k_l / \rho_l h_{fg}) (\partial T / \partial y)_{y=\delta} \quad (12)$$

Further from equation (9) and (10) values of ‘ u ’ and ‘ w ’ can be substituted in equation (12) and solved to obtain $\delta(z, \phi)$ as follows;

$$\left[\frac{2g\rho_l(\rho_l - \rho_v)\cos\theta}{\mu_l * D} \right] (\delta^3 \sin\phi \frac{\partial\delta}{\partial\phi} + \frac{\delta^4}{3} \cos\phi) + \left[\frac{g\rho_l(\rho_l - \rho_v)\sin\theta}{\mu_l} - \frac{\rho_l}{\mu_l} \frac{dP}{dz} + \right] \quad (13)$$

$$\rho_l \tau_i \mu_l \delta \delta \delta \partial \delta \partial z + \rho_l * \delta \mu_l \delta \delta \delta \partial \tau_i \partial z - \delta \delta \delta \partial \delta \partial z dP/dz - [k_l(T_{sat} - T_w)h_{fg}] =$$

0

Thus, a partial differential equation is obtained which can be solved to find value of ‘ δ ’ such that ‘ $\delta = \delta(z, \phi)$ ’, if the value of stratified angle is known which lies within the limits of $0 \leq \phi \leq \phi_s$. Furthermore, to solve the above differential equation, numerical schemes are used as steps mentioned below: -

1. Firstly, at tube inlet, i.e. at $z=0$, the above equation is reduced to ordinary differential equation in ‘ ϕ ’. Which will yield values of $\delta(0, \phi)$ for different values of ‘ ϕ ’ in the respective limit.
2. Secondly for $\phi=0$, at top of tube, the equation in the form of $(\partial\delta/\partial z)$ can be solved using Runge Kutta method. This will give values of $\delta(z, 0)$ for different ‘ z ’.

3. Finally solving equation (13) by using iterative numerical method for provided stratified angle.

D. Solution to find $\delta(0, \varphi)$

At inlet of tube $z=0$, which can be substituted in above equation. Followed by $\partial\delta/\partial z=0$ at this location. Hence now solving equation (13) for the above-mentioned condition, we have

$$\left[\frac{2g\rho_l(\rho_l-\rho_v)\cos\theta}{\mu_l*D}\right](\delta^3\sin\varphi\frac{\partial\delta}{\partial\varphi} + \frac{\delta^4}{3}\cos\varphi) = [k_l\frac{(T_{sat}-T_w)}{h_{fg}}] \tag{14}$$

By solving the above ordinary differential equation, we get,

$$\delta(0, \varphi) = (k_l\frac{2(T_{sat}-T_w)\mu_l*D}{h_{fg}*g\rho_l(\rho_l-\rho_v)\cos\theta})^{0.25}(\sin\varphi)^{\frac{1}{3}} [\int(\sin\varphi)^{1/3} d\varphi]^{0.25} \tag{15}$$

E. Solution to find $\delta(z, 0)$

At the next step to find another boundary condition for solving equation (13), considering top of tube where $\varphi=0$, equation (13) reduces to;

$$\left[\frac{2g\rho_l(\rho_l-\rho_v)\cos\theta}{\mu_l*D}\right]\frac{\delta^4}{3} + \left[\frac{g\rho_l(\rho_l-\rho_v)\sin\theta}{\mu_l} - \frac{\rho_l}{\mu_l}\frac{dP}{dz} + \frac{\rho_l\tau_i}{\mu_l\delta}\right]\delta^3\frac{\partial\delta}{\partial z} + \left(\frac{\rho_l*\delta}{\mu_l}\right)\left[\frac{\delta^2}{2}\frac{\partial\tau_i}{\partial z} - \frac{\delta^3}{3}\frac{\partial}{\partial z}\left(\frac{dP}{dz}\right)\right] - \left[k_l\frac{(T_{sat}-T_w)}{h_{fg}}\right] = 0 \tag{16}$$

F. Pressure drop calculation

The drop in pressure is considered as function of vapour velocity. This can be given as

$$\frac{dP}{dz} = \frac{2f_{fric}u_v^2\rho_v}{d_h} \text{ and } d_h = \frac{4A_v}{P_v}$$

Where ' d_h ' is hydraulic diameter calculated as above

The value of ' A_v ' gives cross sectional area occupied by vapour and ' P_v ' gives vapour phase perimeter. Both the mentioned terms are dependent on stratified angle ' φ_s ' as

$$A_v = \frac{D^2}{8}(2\varphi_s - \sin 2\varphi_s) \text{ And, } P_v = \frac{D}{4}\varphi_s + D \sin(\pi - \varphi_s)$$

G. Analysis of accumulated condensate layer, ($\varphi_s \leq \varphi < \pi$)

Till this section, we analyzed how the condensate layer grows in thickness across length and around circumference of tube. Further this condensate layer flows down due to gravity and can get accumulated in the pipe. Hence a detailed study is required in this section in order to calculate over all heat transfer coefficient across the condenser. Now for analyzing accumulated condensate layer consider Figure 1.b. The y-axis is considered to give the depth of accumulated layer. The stratified accumulated layer is assumed to be formed at $\varphi = \varphi_s$, and is assumed to increase till φ reaches to a value such as $\varphi = \pi$ i.e. at the bottom of the tube. Let ' y_δ ' be depth of accumulated stratified layer, then from Figure 1.b it can be sated as

$$y_\delta = \frac{D}{2}(1 + \cos \varphi_s) \tag{17}$$

H. Averaged and local heat transfer coefficient

As described in the above section, the overall heat transfer coefficient consists of two parts; of which one comes from heat transfer rate of flowing thin condensate layer and other comes from heat transfer rate of accumulated condensate layer. Therefore, the process of heat transfer in these two different conditions must be modelled differently. Thus, considering the thin flowing condensate layer, the local heat transfer rate in this region can be evaluated by

$$h_c(z, \varphi) = \frac{k_l}{\delta(z, \varphi)} \text{ in range of } 0 \leq \varphi \leq \varphi_s \tag{18}$$

Now equation (18) can be used to find local average heat transfer coefficient as follows

$$h_c(z) = \frac{1}{\varphi_s} \int_0^{\varphi_s} h_c(z, \varphi) \tag{19}$$

Now the heat transfer coefficient given by accumulated condensate layer will be obtained by solving;

$$h_1(z) = \frac{k_l}{y_\delta(z)} \quad (20)$$

This is negligibly small when compared to heat transfer coefficient obtained from flowing condensate layer, hence was not considered during past analysis related to condensation phenomenon in tube. Hence the overall local heat transfer coefficient can be now obtained by summing equation (19) and equation (20) while considering effect of stratified angle such as

$$h(z) = \frac{\varphi_s(z)}{\pi} h_c(z) + \left[1 - \frac{\varphi_s(z)}{\pi}\right] h_1(z) \quad (21)$$

III. RESULT AND DISCUSSION

Chen et al [2], has performed analysis on horizontal tube condensation with tube diameter of 25.4mm and tube length of 3m, same case is used for validating the obtained mathematical model with above listed assumptions. They considered the effect of interfacial shear stress and pressure drop across tube. The inlet saturation temperatures were taken to be 60 °C, 70 °C and 100 °C respectively. The wall temperatures were varied in a particular range such that difference between temperatures was obtained as 1 °C and 2 °C respectively. The working fluid was considered to be water vapor (steam) with inlet vapor quality of unity ($x_1=1$).

The Figure 2 gives the variation of stratified angle across length of condenser tube with different temperature difference and inlet saturation temperature of 100 °C with inlet vapor Reynolds number of 30,000. The effect was checked for temperature difference of 1 °C and 2 °C respectively. It could be noted that the variation of stratified angle hence obtained has an error of below 2.2% with that values given in [2] and shows a good agreement when compared with different results obtained from literature. The reason behind the error can be thus explained with the above considered assumptions. Also, it can be thus stated that the stratified angle decreases with increase in temperature difference.

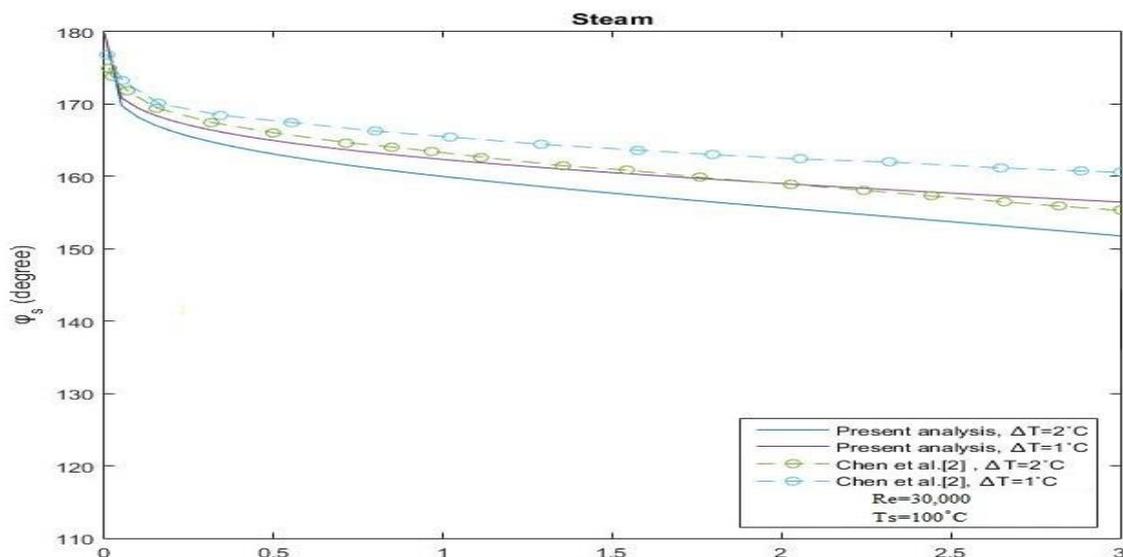


Figure 2 Comparison of stratified angle variation along the tube length at various temperature differences obtained during present analysis with that given by Chen et al. [2]

Figure 3 represents the change in average condensing flowing film heat transfer coefficient given by equation (19) and local average heat transfer coefficient given by equation (21).

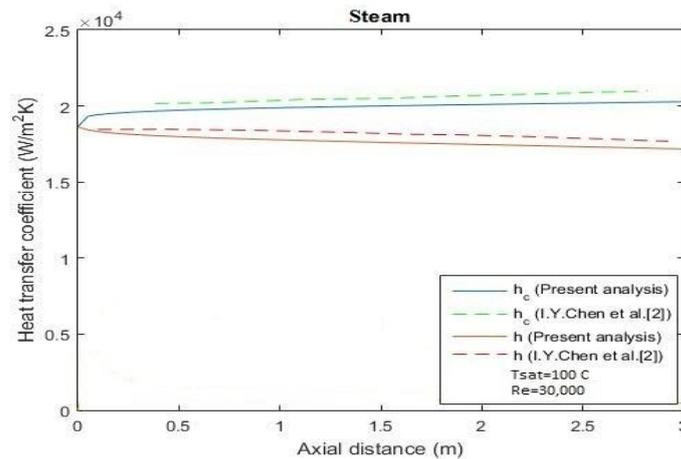


Figure 3 Comparison of axial variation of local averaged heat transfer coefficient obtained in present analysis with that given by Chen et al. [2]

When compared to results that are obtained by I.Y.Chen et al. [2] it can be seen that results obtained from present model are acceptable with error in range of 2.5% to 3.5%. Hence the present model is found to give good agreement between the predicted heat transfer coefficients and with that obtained from [2].

Figure 4 represents the comparison of experimental heat transfer coefficient as obtained by [6] with that of the heat transfer coefficient obtained from developed mathematical model for steam. The result of run1 and run2 (Jack H. G. et al. [6]) is represented in the figure 4. A vertical stainless steel condenser tube was used for experimentation as mentioned in [6]. The error in result obtained by mathematical model can be as a result of the various constraints or the assumption that were considered while developing the same.

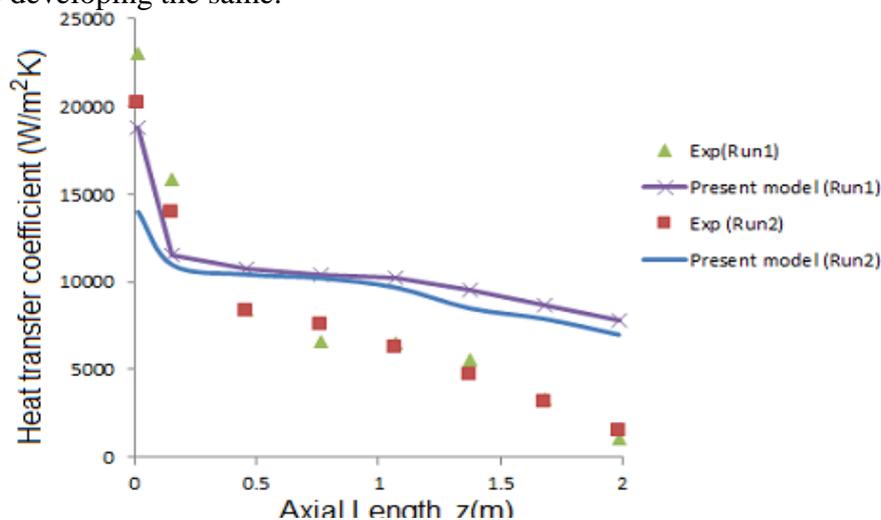


Figure 4 Comparison of condensing heat transfer coefficient obtained by current analysis with that of the results obtained by experimental investigation (Run1 and Run2) by Jack. H. G. et al. [6].

Thus, from all above figures and related discussion, it can be stated that the mathematical model thus obtained is in good agreement with experimental and analytical results of different authors as given in literature.

IV. NOMENCLATURE

ϕ_s = Stratified angle of condensate accumulated layer; τ_i = Viscous shear;
 h_{fg} = Latent heat of condensation; K = Thermal conductivity

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