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FINDING THE INITIAL INTEGER POINT NEAR THE SOLUTION ON A HYPER CIRCLE BY THE RHOMBUS METHOD

Mahammad Maharram Aliyev
National Academy of Sciences of Azerbaijan, Institute of Control Systems, Baku, Azerbaijan

Abstract—In this paper a method for finding the integer point rather close to any hyper plane intersecting the hyper circle and lying on this hyper circle was given. Having accepted this found point as an initial point the (SSP) problem may be solved by the exact, polynomial method. In the method, using two integer points located in different sides of hyper plane intersecting the hyper circle. For constructing the integer point rather close to hyper plane the original rhombus method is used.

Keywords: Knapsack Problem (KP), Subset Sum Problem (SSP), NP-Class, Integer Programming, N-Dimensional Cube, Hyper Plane, Hyper Circle, Hyper Arch

1) M_{nk} – hyper planes with of given equation $x_1 + x_2 + \ldots + x_n = k$; $k = 1,2,\ldots,n-1$
2) O_{nk} - is the center of hyper circle located on hyper planes M_{nk} and holding C_n^k number vertexes points n-dimensional cube.
3) r_{nk} –is radius hyper circle.
4) K_{nk} – hyper circle located on M_{nk} of a radius r_{nk} and center at the O_{nk} .
5) $X(n,k)$– points are such that their k number coordinates consist of a unit, n-k number co-ordinates consist of zero.
6) Restriction functional- $F(W,X)=w_1x_1 + w_2 x_2 + \ldots + w_n x_n$

I. INTRODUCTION

The knapsack problem can be formulated as a solution of the following linear integer programming formulation:

$$(KP)\ \text{maximize} \sum_{j=1}^{n} p_j x_j \qquad (1.1)$$

$$\text{subject to} \sum_{j=1}^{n} w_j x_j \le c \qquad (1.2)$$

$$x_j \in \{0,1\},\ \ j = 1,\ldots,n$$

When $p_j = w_j$ in (KP), the resulting optimization problem is known as the subset sum problem (SSP) because we are looking for a subset of the values w_i with the sum being as close as possible to, but not exceeding the given target value c .

$$(SSP)\ \text{maximize} \sum_{j=1}^{n} w_j x_j ;$$

$$\text{subject to} \sum_{j=1}^{n} w_j x_j \le c$$

$$x_j \in \{0,1\}, \quad j = 1,\ldots,n$$

Although (SSP) is a special case of (KP) it is still NP-hard and is NP - complete [1].

The theory of NP - completeness gives us a framework for showing that it is very doubtful that a polynomial algorithm exits. Indeed, if we could find a polynomial algorithm e.g. for solving the subset sum problem, then we would also be able to solve numerous famous optimization problems like the travelling salesman problem, general integer programming [1].

By solving first the NP class (SSP) problem by the exact, polynomial, determined method, the following scheme was used [2,3,4].

- the geometrical interpretation of n-dimensional cube was given and the set of vertices point of this cube being the domain of definition of this problem was grouped on K_{nk} hyper circles located on $n-1$ number parallel to M_{nk} hyper planes. The local solution of the problem on each hyper circle was found and the general solutions were chosen from these local solutions.
- interrelation constraint hyper planes and hyper circles was studied.
- local properties of arbitrary point on the hyper circle were determined.
- the method for constructing a sequence approximating to the solution on the hyper circle was shown
- the theorem on necessary and sufficient conditions for local optimal solvability of a point on the hyper circle was proved.
- the number of iterations was estimated and a method for decreasing their number was determined.
- the difficulty order of the algorithm was studied and its polynomial character was shown.

II. PROBLEM STATEMENT

For solving the (SSP) problem by an exact, polynomial method, we choose the initial point near the constraint hyper planes on the hyper arch. When this point satisfies the on condition that $c - (W, X(n,k)) \le n$, the polynomial character of the algorithm is provided. In [2,4], for finding this point, it is necessary to solve the system of equations:

$$\begin{cases} w_1 l_1 + w_2 l_2 + \ldots + w_n l_n = 0 \\ l_1 + l_2 + \ldots + l_n = 0 \end{cases}$$

Here, giving free values to the $n-2$ unknown, satisfying the on condition that $w_1 \neq w_k$, we determine l_1 and l_k according to these values.

Not solving the system of indefinite equations with finite solution we can solve this problem by a more convenient method.

The goal of the paper is to describe a method for finding the integer point close to arbitrarily hyper plane intersecting the hyper circle, and located on the hyper circle.

III. FINDING THE INTEGER POINT CLOSE TO THE SOLUTION ON A HYPER ARCH BY THE RHOMB METHOD

Assume that the space is n-dimensional, k is the ordinal number of the hyper circle. $k=\overline{1,n-1}$. The k number coordinate of the integer point located on K_{nk} equals a unit, $n-k$ number coordinate equals zero.

$$n_1 = init(n/2); \quad k_1 = init(k/2).$$

Order the elements of the vector $W(w_1, w_2,..., w_n)$ by increasing order and denote it by WS: Let

$$ws_1 \le ws_2 \le ... \le ws_n; \quad q = n_1 - k_1$$

$$q+1 \le j \le q+k \; ; \; i=1,2,...,n$$

$$\text{If } w_i - ws_j = 0 \text{ then } xo_i = 1$$

$$\text{If } w_i - ws_j \ne 0 \text{ then } xo_i = 0.$$

Verify if the point $XO(n,k)$ determined in such a way satisfies the constraint and if it is close to the solution hyper planes

$$(W, XO) \le c \tag{3.1}$$

$$c - (W, XO) \le n. \tag{3.2}$$

If condition (3.1) is satisfied, condition (3.2) is not satisfied, we replace this point by a point satisfying condition (3.1) and (3.2).

Determine the integer points where the constraint functional on the hyper circle takes the greatest and least values. Denote the k-th maximum element of the vector WS by $WS_{k\max}$. To x_j corresponding to the k number w_j satisfying the condition $ws_{k\max} - w_j \le 0$ give the value a unit, to the other $n-k$ number x_j the value zero, and denote the obtained point by $X_{\max}(n,k)$. This point is the integer point giving the greatest value to the constraint functional on K_{nk}.

Denote the k-th minimum of the vector WS by $ws_{k\min}$. By giving to the x_j corresponding to the k number w_j satisfying the condition $ws_{k\min} - w_j \ge 0$ a unit value, to x_j corresponding to $n-k$ number w_j the zero value, we can determine the point $X_{\min}(n,k)$. This point is the integer point giving the least value to the constraint functional on the hyper circle K_{nk}.

Now, show the rule for finding an integer point rather close to any hyper plane intersecting the hypercircle and to an arbitrary point located on the hyper circle by the rhomb method.

We verify if the point $XO(n,k)$ satisfies the condition $(W, X) \le c$ (3.1). If the point $XO(n,k)$ satisfies also condition (3.2), the needed conditions are satisfied and accepting this point as an initial point for the exact method described in [4], at the edge of the hyper arch we can find the exact solution.

If only (3.1) is satisfied, then we find the vectors

$$V_1 = O_{nk} - X_{\max}(n,k) \text{ and } V_2 = O_{nk} - XO(n,k).$$

O_{nk} is the center of the k -th hyper circle

$$V = V_1 + V_2 \ ; \ |V_1| = |V_2| = r_{nk}.$$

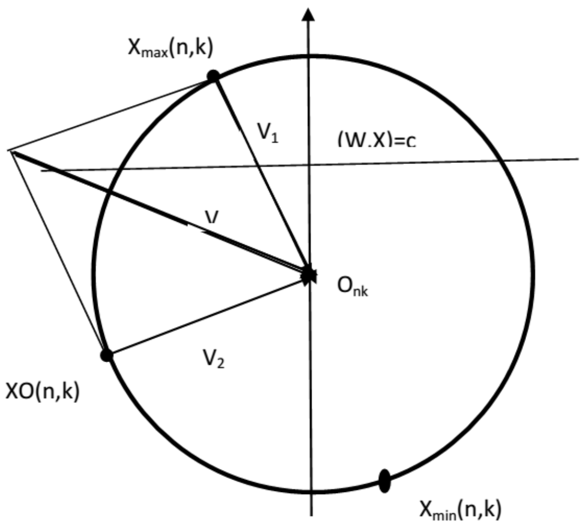

Figure 1. Illustration of the romb rule

The vector V is the diagonal of the rhombus with the sides V_1 and V_2 (fig. 1). The diagonal of the rhombus bisects its inner angle and appropriate arch of the circle. By ordering the elements of the obtained vector V by increasing order, we can find the integer point closest to intersection point of the straight line passing through the point O_{nk} in the direction of vector V on the hyper circle [3,4].

If the point $XO(n,k)$ satisfies none of the conditions (3.1) and (3.2), then

$$V_1 = O_{nk} - X_{\min}(n,k) \text{ and } V_2 = O_{nk} - XO(n,k)$$

and we find the vectors $V = V_1 + V_2$.

By ordering the elements of the obtained vector in increasing series, we can find the integer point closest to the intersection point of the straight line passing from the point O_{nk} and intersecting the hyper circle in the direction of the vector V. Denote this point- $X_1(n,k)$. Verify if the point $X_1(n,k)$ satisfies conditions (3.1) and (3.2). If these conditions are not satisfied, we continue the process in the following way. Each iteration we choose the closest point at different sides of the hyper planes, $(W,X) = c$. Define the vector $V^{(q)}$ and according to them construct a sequence of the points $X_2(n,k), X_3(n,k), \ldots, X_q(n,k)$ due to these vectors. If any of these points satisfies the conditions (3.1), (3.2), the process ends. It is clear that the number of operations in this process is polynomial.

We must find an initial point on another semi-circle as well [4].

Finding the intersection point of the straight line passing through the points $W_{\min}$ and $W_{\max}$ and the hyper plane $(W, X) = (W, X_q(n,k))$ we denote it by T .

$$W_{\min} = O_{nk} - r_{nk}WP_n; \; W_{\max} = O_{nk} + r_{nk}WP_n \qquad [4]$$

we find the vector $D = T - X_q(n,k)$

We can determine the initial point of the hyper circle an another semicircle as follows

$$X' = T + T - X_q(n,k) = 2T - X_q(n,k).$$

Finding the integer point closest to the point X' we denote it by $X'(n,k)$ [3,4]. Accepting the point $X_q^{'}(n,k)$ as an initial point, we apply the exact algorithm on the same semicircle and find the solution of the problem on this semicircle.

IV. CONCLUSIONS

In the paper we obtain the following results:

1) The point $XO(n,k)$ giving approximate mean value to the functional $F(X) = (W, X)$ on the hyper circle was found.
2) The integer $X_{\min}(n,k)$ giving to the functional $F(X) = (W, X)$ on the hyper circle the least value and the integer $X_{\max}(n,k)$ giving the greatest value to the functional was determined.
3) Using these points, by the rhombus method, they way for finding the point $X_q(n,k)$ rather close to the hyper plane $(W, X) = c$ and satisfying the restraint was given.
4) A way for finding the point $X_q^{'}(n,k)$ rather close to the solution on another semi hyper circle was shown.
5) Using these results by finding $2n-1$ number suboptimal local solution, among them one can find the suboptimal solution of (KP) and (SSP) problems.

REFERENCES

[1] Hans Kellerer, Urich Pferschy, David Pisinger. Knapsack Problems. Springer-Verlag Berlin.Hidelberg, 2004, 525 p.
[2] Aliyev M.M.One approach to the solution of the knapsack problem//Reports NAS of Azerb.,2005, p. 32-39.
[3] Aliyev M.M. On Solution Method of the Knapsack Problem. Proceedings of the Sixth International Conference on Management Science and Engineering Management. Volume I, Springer-Verlag, London, 20012, p.257-267.
[4] Mahammad Maharram Aliyev. Exact, Polynomial, Determination Solution Method of the Subset Sum Problem. SciencePG. Applied and Computational Mathematics. Vol. 3,No. 5, 2014, pp. 262-267.