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Comparison of ARMAX Model Identification Results Based on Least Squares Method

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Abstract—In this paper, process simulator which carried out wireless temperature control was modelled with linear parametric single input-single output (SISO) model by using the system identification technique. To achieve the data transfer between computer in Process Control Laboratory and the process simulator in Unit Operations Laboratory, wireless communication system established and wireless experiments were performed on-line by means of MATLAB/Simulink program. Wireless data transfers during the experiments were carried out using radio waves at a frequency of 2.4 GHz. ARMAX model parameters were calculated by using three least squares parameter estimation methods including heater capacity and heater output temperature were selected as input and output, respectively. Square wave signal was performed as input signal and model parameters were estimated by using Recursive Least Squares (RLS), Filtered Least Squares (FLS) and Extended Least Squares (ELS) methods by input-output values were collected from the system. In order to the test the model validation, two successive step changes were applied and heater output temperature changes compared with the obtained values from these three least squares methods. According to the results best representing ARMAX model of the system was determined with FLS method and results of RLS and ELS methods close to each other.

Keywords—System identification, MATLAB/Simulink, ARMAX model, parameter estimation, least square method, wireless measurement and control

I. INTRODUCTION

System identification is an important approach to modeling dynamic systems and has been used any areas such as chemical processes, signal processing [1-3]. System identification is an significiant part of any control design and deals with the problem of building reliable mathematical models of dynamic processes. By using system identification tools, one is able to obtain mathematical models of process based on observed input-output data. The system identification results increased the control performance [4]. Several methods have been developed for system identification, e.g., the least squares methods [5-6], gradient based methods [7], the maximum likehood methods [8] and the step response based method [9]. The least squares (LS) method has since been the most popular algorithm for parameter estimation due to its simplicity in concept and convenience in implementation. LS method has been developed under different applications such as discrete and continuous-time approaches, linear and nonlinear systems [10].

The ARMAX model is a widely used model to approximate the practical dynamic system and is one of the most important models in time series analysis as well as in system and control. ARMAX identification technique exist two categories which are time domain and frequency domain approach. There have been appearing a large number of works on ARMAX model order determination and parameters estimation [11]. The order of ARMAX model can be determined by the Akaike Information Criterion (AIC). The estimation of parameters is the most important part in the identification of ARMAX model. ARMAX model estimated methods minimize a criterion function numerically using gradient methods.

The rapid development of wireless technology plays extremely important roles in measurement and control related applications nowadays. In recent years, the demand for wireless communications in many monitoring and measurement applications has grown tremendously, such as agriculture [12], environmental [13], health monitoring [14], logistics [15], industrial [16], commercial, and, etc. Some new technologies like Zig-Bee [17], Wi-Fi, and Bluetooth have already made significiant contribution to data acquisition. As the same time, they arise some new challenges to guarantee a highly reliable, accurate and fault tolerant process. It is a critical issue to develop innovative approaches to deal with multi-variable, multi space problem domains (detection, identification, tracking, data fusion, energy-efficiency and fault tolerant framework) as well as practical implementation in wireless measurement and control application.

The purpose of this paper is wireless temperature control of a process simulator was modelled with linear parametric SISO model by using the system identification technique. Wireless experiments were performed on-line by means of MATLAB/Simulink program and wireless data transfers during the experiments were carried out using radio waves at a frequency of 2.4 GHz. ARMAX model parameters were calculated by using three least squares parameter estimation methods including heater output temperature and heat of heater were selected as output and input, respectively. In order to the test the model validation, step change response was used and heater output temperature changes compared with the obtained values from these three methods.

1. Developing ARMAX Model for A SISO Process

Linear model on the discrete time for a SISO process given in Eq. (1);

$$y(t) = G(z^{-1};\theta)u(t) + H(z^{-1})e(t) \quad \textbf{(1)}$$

In Eq. (1) *y(t)* is output variable with *na* dimension, *u(t)* is input variable with *nb* dimension and *e(t)* is the load which composed random values. Eq. (2) which is a linear model structure is obtained if Eq. (1) is shown with *A*, *B*, *C*, *D* and *F* polynomials.

$$A\left(z^{-1}\right)y(t) = \frac{B\left(z^{-1}\right)}{F\left(z^{-1}\right)}u(t) + \frac{C\left(z^{-1}\right)}{D\left(z^{-1}\right)}e(t) \quad \textbf{(2)}$$

ARMAX model is shown in Eq. (3), in the case of *F* and *D* polynomials are one on this model;

$$A\left(z^{-1}\right)y(t) = B\left(z^{-1}\right)u(t) + C\left(z^{-1}\right)e(t) \quad \textbf{(3)}$$

A, B and *C* are also polynomials in the backward shift operator z^{-1} given in Eq. (4);

$$\begin{aligned} A\left(z^{-1}\right) &= 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_{na} z^{-na} \\ B\left(z^{-1}\right) &= b_1 z^{-1} + b_2 z^{-2} + ... + b_{nb} z^{-nb} \\ C\left(z^{-1}\right) &= 1 + c_1 z^{-1} + c_2 z^{-2} + ... + c_{nc} z^{-nc} \end{aligned} \quad \textbf{(4)}$$

ARMAX model, is written as the difference equation form given in Eq. (5);

$$\begin{aligned} & y(t) + a_1 y(t-1) + ... + a_{na} y(t-na) = b_1 u(t-1) + ... + b_{nb} u(t-nb) + e(t) + \\ & c_1 e(t-1) + ... + c_{nc} e(t-nc) \end{aligned} \quad \textbf{(5)}$$

If Eq. (5) written in matrix form, system parameter vector and data vector is obtained with Eq. (6) and Eq. (7), respectively;

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$$\theta^T = [a_1........a_{na} \;\; b_1........b_{nb} \;\; c_1.......c_{nc}] \tag{6}$$

$$\varphi^T = [-y(t-1)........-y(t-na) \;\; u(t-1)........u(t-nb) \;\; e(t-1).........e(t-nc)] \tag{7}$$

Thus, the Eq. (8) obtained which output variable is connecting to the data vector and the parameter vector;

$$\hat{y}(t) = \varphi^T(t)\hat{\theta} + e(t) \tag{8}$$

where the superscript T denotes the matrix transpose given in Eq. (8) is a linear equation due to relationship between $\varphi^T(t)$ and θ parameters and is a start point of parameter calculation algorithms.

II. DETERMINED ARMAX MODEL PARAMETERS WITH LEAST SQUARES METHOD

Fitting a process model to an experimental data, the main objective is to obtain estimates of parameters of interest, such that the differences between predictions and observations (residuals) are minimal. This is usually accomplished by minimizing a "sum of squares" objective function. Three of the commonly used least squares methods are Recursive Least Squares (RLS), Filtered Least Squares (FLS) and Extended Least Squares (ELS) approach.

1. Recursive Least Squares (RLS)

RLS is one of the best method for system identification. Compared to mean least squares (MLS) algorithms, RLS algorithms have a faster convergence speed and do not exhibit the eigenvalue spread problem. However, RLS algorithms involve more complicated mathematical operations and require more computational resources than LMS algorithms. In this method, system model parameters are recalculated using new measurement data for each sampling time. An algorithm for RLS given below;

At (t+1) time,

i. $\varphi(t+1)$ vector is formed with using new input $u(t+1)$ and output $y(t+1)$ variables data,

ii. $\varepsilon(t+1)$ estimation error is calculated with Eq. (9) ;

$$\varepsilon(t+1) = y(t+1) - \varphi^T(t+1)\hat{\theta}(t) \tag{9}$$

iii. *P(t+1)* covariance matrix is calculated with Eq. (10) ;

$$P(t+1) = P(t) - \frac{P(t)\varphi(t+1)\varphi^T(t+1)P(t)}{1+\varphi^T(t+1)P(t)\varphi(t+1)} \tag{10}$$

iv. Parameter vector is updated with Eq. (11) ;

$$\hat{\theta}(t+1) = \hat{\theta}(t) + P(t+1)\varphi(t+1)\varepsilon(t+1) \tag{11}$$

v. Wait until the next time and then return to the first step.

2. Filtered Least Squares (FLS)

In the FLS method, data vector (φ^T) which formed input and output variables is filtered with a suitable polinomial. Eq. (12) is obtain with filtered input and output variables are shown u_f and y_f, respectively;

$$Ay_f(t) = Bu_f(t-1) + e(t) \tag{12}$$

In this case data and parameter vector given in Eq. (13) and (14), respectively;

$$\varphi^T = [-y_f(t-1).....-y_f(t-n)\ \ u_f(t-1).....u_f(t-n).....u_f(t-m-1)] \quad (13)$$

$$\theta^T = [a_1.......a_n\ b_0......b_m] \quad (14)$$

In this study data vector was filtered through a polynomial given in Eq. (15);

$$C(z^{-1}) = (1 + 0.1z^{-1} + 0.04z^{-2}) \quad (15)$$

3. Extended Least Squares (ELS)

In the ELS algorithm, noise parameters such as noise variances are also calculated. Estimation for coefficients of C polinomial in Eq. (3) values of *e(t-1), e(t-2),.....,e(t-nc)* must be known. In practice *e(t)* values can not be measured and error of estimation $\varepsilon(t)$ which is calculated with Eq. (16) used instead of *e(t)* values.

$$\varepsilon(t) = y(t) - \varphi^T(t)\hat{\theta}(t-1) \quad (16)$$

An Algorithm for Extended Least Squares (ELS) given below;

At (t+1) time;

i. Vector of $\varphi(t+1)$ is formed using new input $u(t+1)$ and output $y(t+1)$ variables data,
ii. Estimation error is calculated with Eq. (9) ;
iii. *P(t+1)* covariance matrix is updated with Eq. (10) ;
iv. Parameter vector is updated with Eq. (11) ;
v. Based on Eq. (16), the following extended vectors are introduced with Eq. (13) and Eq (14) ;
vi. Wait until the next time and then return to the first step.

III. EXPERIMENTAL SYSTEM AND PROCEDURE

The process control simulator consists of two main units, an instrument console and a framework carrying the process equipment which is shown in Figure 1. The instrument console contains the electronic flow, level, temperature controllers and electrical switchgear. It is connected to the process equipment by several cable assemblies. The process equipment consists of a water tank, water circulating pump, electrical water heater, two vessels, two electrically positioned control valves and a heat exchanger. In process control simulator, twelve manual valves are available for different process experiment loops. In the process simulator, temperature measurement and control can be made at four different points which are first tank (T1), heater output (T2), second tank input (T3) and second tank output (T4).

The wireless system developed for transferring data between the computer and the control panel. To achieve the data transfer between computer in Process Control Laboratory and the process simulator in Unit Operations Laboratory, by using the two antennas are found in the laboratory connected to the computer and outside connected to the process simulator. Control valves outputs are connected to the modules, the necessary calibrations are made. The water is pumped via the electrical heater into the reactor up to a certain level. The water then flows back to the sump tank via the cooler. Heat is fed to the water by the heater and residual heat removed by the cooler so as to return the sump tank water temperature to a suitable base level. Heater which is connected on-line to the computer is used as a manipulated variable [18].

Wireless temperature experiments were achieved by using MATLAB/Simulink program and wireless data transfer during the experiments were carried out using radio waves at a frequency of 2.4 GHz. MATLAB/Simulink block diagram was used for the on-line wireless temperature experiments with a computer in the office. There have four moduls, wireless on/off block for the wire or wireless experiments, numerical or graphical display blocks of process parameters, blocks of giving numerical values of valve openness, blocks of stored errors on the MATLAB/Simulink block diagram shown in Figure 2.

The wireless data generated by operating the process simulator described above and shown in Figure 1. Wireless data were used for the development of the three ARMAX models based on least squares method and applied on the process simulator using Process Identification Technique. Wireless temperature experiments were carried out on process simulator during the 1500s time period. First 300s the heater operated % 10 heating capacity for the temperature is expected to become at steady-state. The fluid flow was obtained by running the pump when the level control valve was % 40 opening. The cooling water was opened after liquid level was fixed value. The heater output temperature was expected to become at steady-state while % heating capacity was on. Square wave effects were performed as input signal which apply the heater capacity between %10-90 values and the heater output temperature changes with time as output signal. This square wave effects were given to the heater and heater output temperature data taken with MATLAB/Simulink during the experiments shown in Figure 2 [18].

ARMAX model parameters were calculated by using three least squares parameter estimation methods including heater output temperature and heater capacity values were selected as output and input data, respectively. Square wave signal was performed as input signal and model parameters were estimated using three least squares methods (RLS, FLS, ELS) by input-output values were collected from the system.

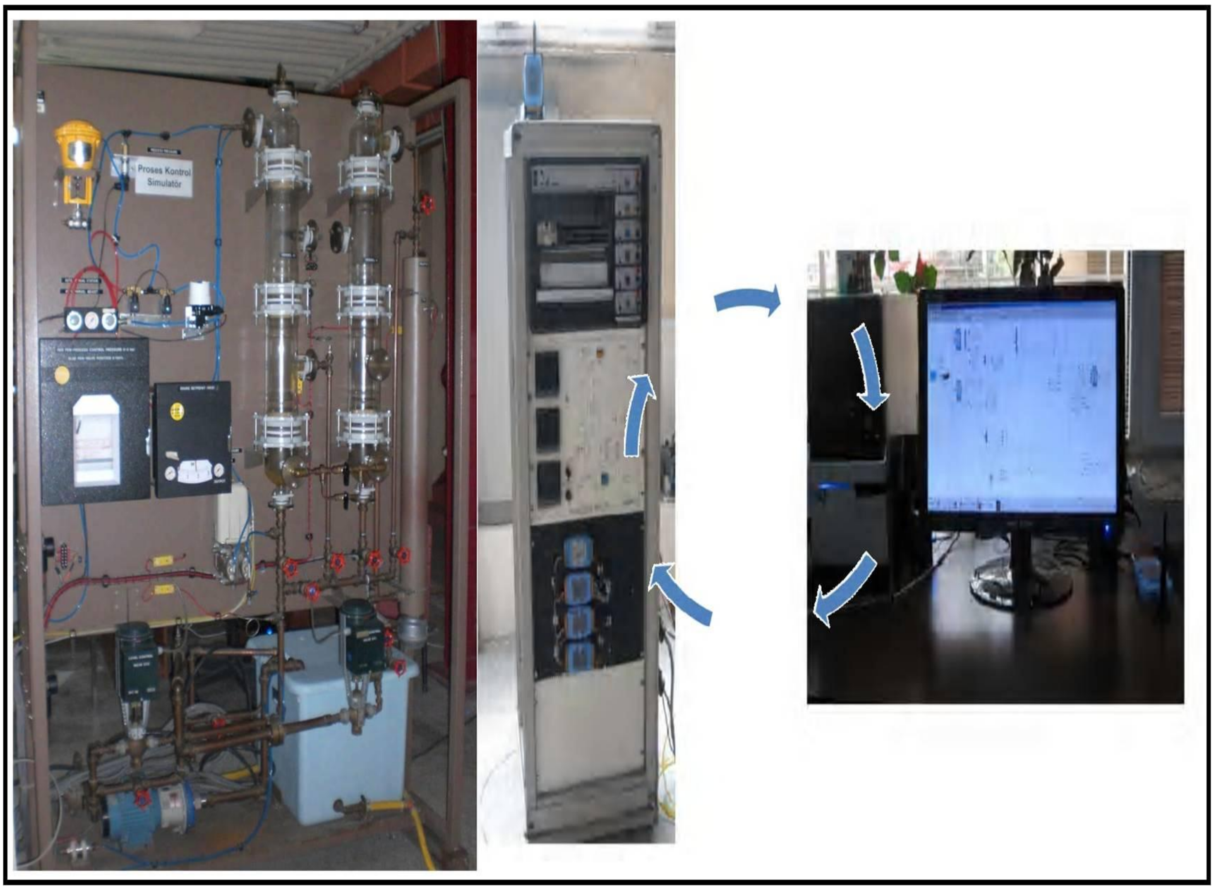

Figure 1. Experimental system: Process simulator, control panel and computer on-line connected to the process simulator with wireless technology [18]

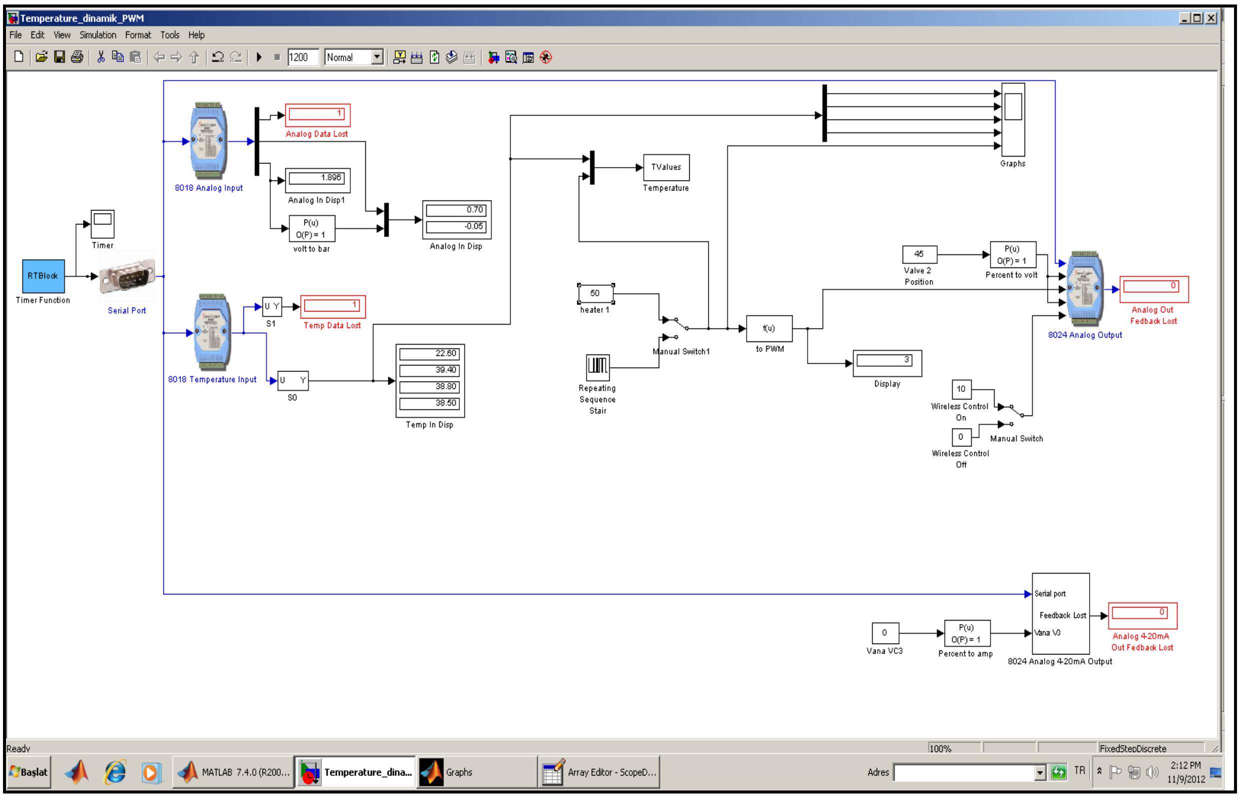

Figure 2. MATLAB/Simulink block diyagram for wireless temperature experiments [18]

IV. RESULTS AND DISCUSSION

Square wave effects were performed as input signal which applied the heater capacity between %10-90 values and the heater output temperature changes with time as output signal shown in Figure 3. ARMAX type linear parametric models and model parameters of a process simulator were calculated with three least squares methods for the system identification which significant part of the model predictive control. The models which used in design of the process selected to the best represent the process and model parameters calculated the correctly for the control systems works effective and unblemished. For this purpose, based on the least squares method second order linear ARMAX models were calculated given in Table 1.

Table 1. Obtained ARMAX models based on Least Square Methods

Method	ARMAX Model
R.L.S.	$(1-1.97z^{-1}+0.9698z^{-2})y(t)=(-0.003433z^{-1}+0.003569z^{-2})u(t)$ $+(1-0.7014z^{-1}-0.2411z^{-2})e(t)$
F.L.S.	$(1-1.027z^{-1}+0.02578z^{-2})y(t)=(-0.0004195z^{-1})u(t)$ $+(1-0.1073z^{-2})e(t)$
E.L.S.	$(1-1.951z^{-1}+0.9508z^{-2})y(t)=(-0.0001988z^{-1}+0.0002407z^{-2})u(t)$ $+(1-1.058z^{-1}-0.1071z^{-2})e(t)$

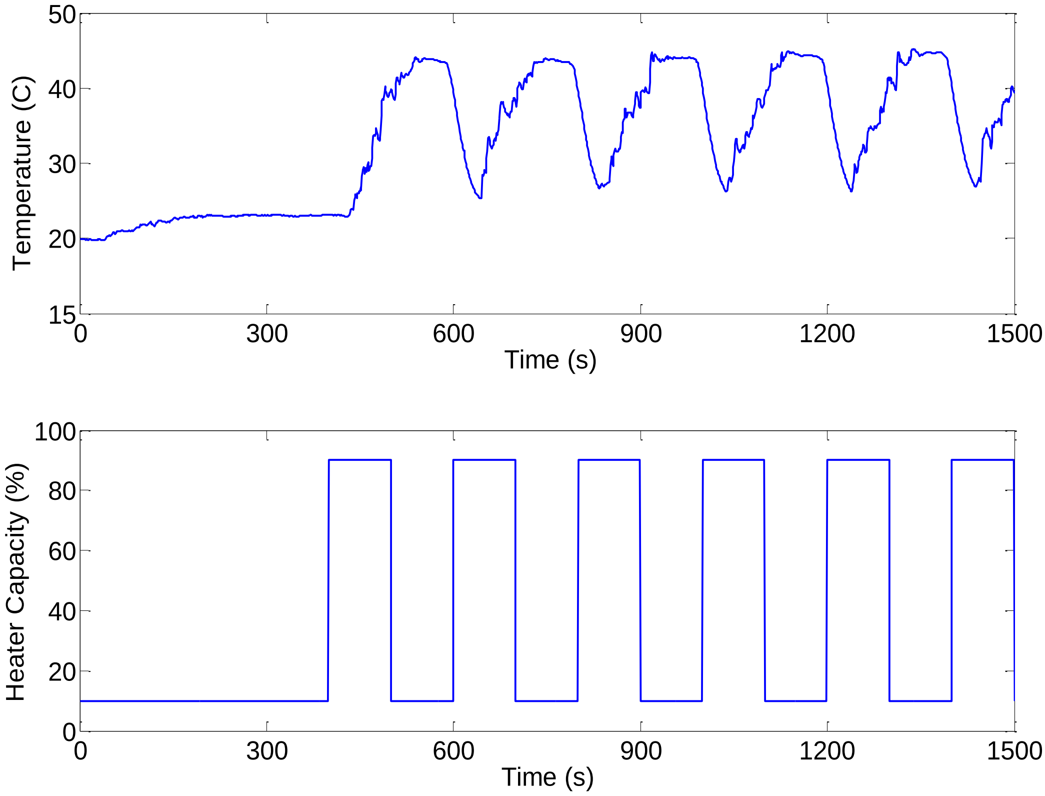

Figure 3. Heater output temperature changes with a square wave effect to the heater capacity between % 10-90 values

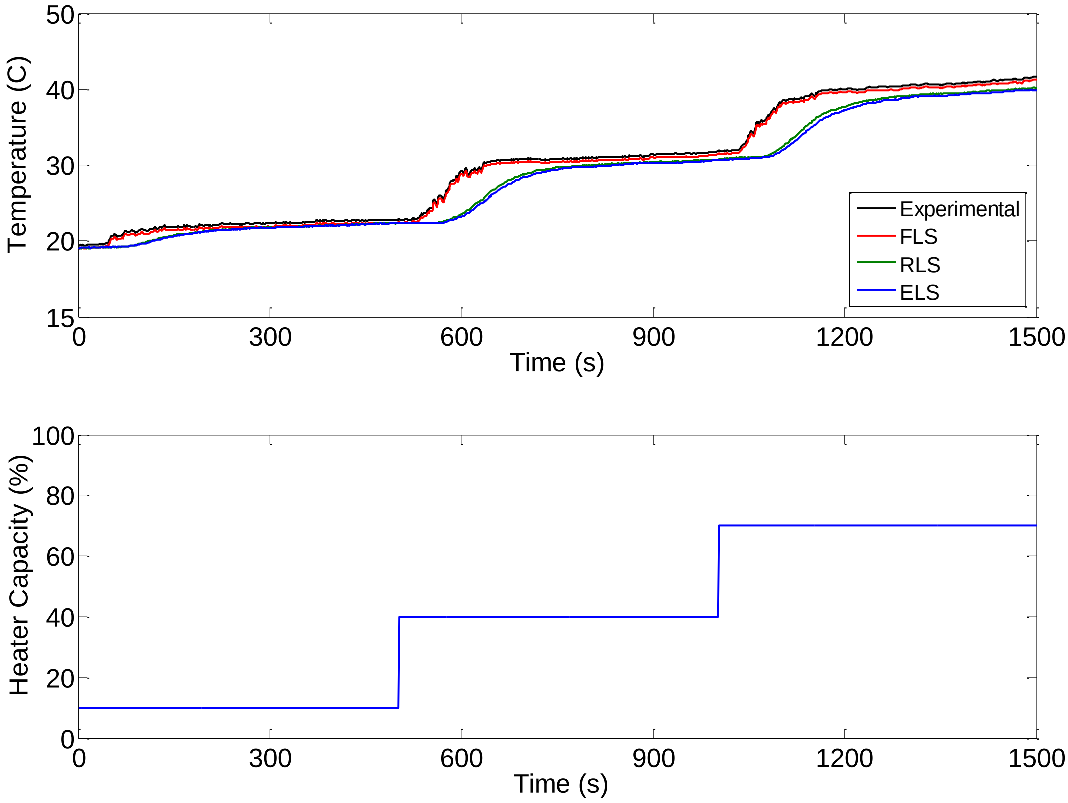

Figure 4. Response of the ARMAX models obtained by three least squares method

For choose the best suitable ARMAX model of system which were determined with the three least square methods, give two successive step changes to the heater between %10-40-70 values and heater output temperature changes compared with the obtained values from these three methods shown in Figure 4. According to the Figure 4, the best representing ARMAX model of system was determined with FLS method. ARMAX model results which were applied with RLS and ELS methods close to each other. According to the ARMAX model results, response of system was faster with FLS method than the RLS and ELS method.

V. CONCLUSION

In this study, process simulator which carried out wireless temperature control was modelled with linear parametric SISO model by the three different ARMAX models based on least square methods. Wireless experiments were performed on-line by means of MATLAB/Simulink program and wireless data transfers during the experiments were carried out using radio waves at a frequency of 2.4 GHz. ARMAX model parameters were calculated by using three least squares parameter estimation methods including heater output temperature and heater capacity were selected as output and input, respectively. Square wave effects were applied the heater capacity and model parameters were estimated using RLS, FLS, ELS methods by input-output values were collected from the system. In order to the test the model validation, step change response was used and heater output temperature changes compared with the obtained values from these three methods. According to the results best representing ARMAX model of system was determined with FLS method and results of RLS and ELS methods close to each other.

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Nomenclature and Synonyms

A,….,H	Polynomials in the discrete time input/output model
A/D	Analog/Digital transducer
ARMAX	Auto Regressive Moving Average with Exogeneous
D/A	Digital/Analog transducer
e(t)	Noise
P(t)	Covariance matrix
t	Time (s)
T	Temperature
u(t)	Input variable
u_f (t)	Filtered input variable
y(t)	Output variable
y_f (t)	Filtered output variable
$y(t)$	Output variable obtained from model
z	Reverse scroll function (t-1), (t-2)
ε(t)	Difference between real output value and model output value
θ	Vector of parameter
$\hat{\theta}$	Vector of calculated parameter
θ^T	Transpoze of parameter vector
φ	Vector of data
φ^T	Transpoze of data vector

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