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An EPQ Model with Time Dependent Demand Rate and Imperfect Production

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Abstract-In this inventory model production process are taken as single-stage where the defective items are produced regularly with a proportion of defectives as scrap. Defective items are remanufactured after the regular production. Remanufactured items are good items and serviceable. Production rate is constant and demand is time dependent. This paper includes a model without shortages. Numerical example is presented to illustrate the models. Special cases of the models are also discussed.
Keywords-Time dependent demand, Perfect inventory, Imperfect inventory Remanufacturing Process.

I. INTRODUCTION

Inventory models for constant demand rate have been considered by many researchers in the past. But usually the demand rate is a variable. Demand of items may vary with time. In recent years, there is a spate of interest in the problem of finding the economic replenishment policy for an inventory system having a time-dependent demand pattern. It started with the work of Silver and Meal[1], who developed a heuristic approach to determine EOQ in the general case of a time-varying demand rate. Donaldson [2] came out with a full analytic solution of the inventory replenishment problem with a linear trend in demand over a finite-time horizon. Manna and Chaudhuri[3] developed an economic order quantity model with ramp type demand. Panda et al [4] developed an inventory model for perishable seasonal products with ramp-type demand. Wee et al [5]. proposed an integrated model for deteriorating items in which shortages were completely back-ordered. Wee et al [6] considered permissible shortage backordering and the effect of varying backordering cost values. Production processes are sometime imperfect. In an imperfect production process, the defective items are produced with perfect finished goods. When raw materials are processed through a production process into finished goods, three types of finished products can be delivered due to various production qualities and material defects. These are (a) quality of perfect finished products, (b) defective products which can be reworked, and (c) scrap. The defective items can be reworked, if no facilities of rework process exists, all defective products go to scrap or sold at reduced prices incurring additional cost as well as the loss of goodwill if the company fails to meets demands of customers. Therefore rework process is necessary to convert the defectives into perfect finished goods. Cheng [7] developed an EOQ model with demand dependent unit cost and imperfect production processes. Hayek and Salameh [8] assumed that all defective items produced were repairable and determined an optimal point for an EPQ model including the reworking of imperfect quality items. Chiu [9] presented a finite production model with a random defective rate- scrap, the reworking of repairable defective items and backlogging. Jamal et al. [10] considered a single production system with rework options and incorporated two rework processes to minimize total system cost. Barzoki et al. [11] studied the optimal run time problem of an EPQ model with imperfect products, the reworking of the repairable defective products and the rejection of non rework able defective items. Cardenas-Barron[12] developed an EQP model with backorders to determine the EPQ for a single product manufactured in a single stage manufacturing system that produced imperfect quality products with the defective products being reworked in the same cycle. Krishanmoorthi and Panayappan[13] and Krishanmoorthi and Panayappan[14] developed production inventory models in an imperfect production environment where defective items are produced with a proportion of scrap. Production rate and demand rate both were assumed to be constant. Krishanmoorthi and Panayappan[13] developed model without shortages while Krishanmoorthi and Panayappan[14] developed model with shortages.

II. MODEL WITHOUT SHORTAGES

2.1 Assumptions and Notations

The following assumptions and notations are applied in the proposed model:

Assumptions:

(1) The item is a single product; it does not interact with any other inventory items.

(2) The regular production rate P is always greater than the sum of the demand rate and the rate at which defective items are produced.

(3) In the remanufacturing process production rate of reworking defectives items is the same as that the regular production rate P.

(4) The demand rate is linear i.e. $D(t) = (a + bt)$ where a, b are positive constants and $a > b$.

(5) Production of defective items is constant and only one type of defective is produced in each cycle.

(6) A certain fraction of defective items produced at the production process is scrap and cannot be reworked so it is disposed off and the remaining fraction except scrap of defectives are reworked and all reworked items are good and serviceable as the original one.

(7) The reworking process is performed after the regular process is finished.

(8) The total good items produced in order to meet the demand are from the regular production and reworking process.

Notations:

(1) T is the infinite length of the cycle.

(2) P is the constant production rate of regular production.

(3) x is the proportion of defective items from regular production.

(4) θ is the proportion of defective items which is scrap.

(5) A is the setup cost.

(6) Q is the production quantity.

(7) Q_d is the quantity of defective items.

(8) Q_s is the quantity of scrap items.

(9) C_h is the inventory holding cost per unit per unit time for perfect and imperfect items.

(10) C_p is the production cost per unit.

(11) C_r is the reworking cost per unit of defective items.

(12) C_d is the disposal cost per unit.

(13) C_q is the screening cost per unit

(14) $I(t)$ is the inventory level of perfect items at any time t in the cycle $[0,T]$.

(15) $I_1(t)$ is the inventory level of imperfect items at any time t in the interval $[0,t_2]$.

(16) t_1 is the time at which regular production stops and defective items are started to rework with the production rate *P*.

(17) t_2 is the time at which reworking process of defective items is completed.

(18) $TC(t_1)$ is the total cost per unit time.

2.2 Formulation and Analysis of the Model

During the production cycle $[0,T]$, time intervals $[0,t_1],[t_1,t_2]$ and $[t_2,T]$ are considered separately. During the time period $[0,T]$ the demand is linearly time dependent. At time $t=0$, production starts at the rate *P*. In this regular production process defective items are produced at a rate xP where $0<x<1$. A fraction θ of these defectives is scrap and disposed off simultaneously. Thus perfect items produced at a rate $P-xP$. This rate $P-xP$ is greater than the demand rate $D(t)=a+bt, a>b$. Therefore up to time t_1 inventory level is increased. At time t_1 the regular production stops and the accumulated defective items are started to rework with the production rate *P*. In the time interval $[t_1,t_2]$ inventory level is again increased. At time t_2, reworking process is completed. Now due to market demand inventory level starts to decrease and reaches zero at time *T*. At time *T* the next cycle begins.

The formulation of the model for imperfect inventory level $I_1(t)$ of imperfect items in the time interval $[0,t_2]$. At time $t=0$, the inventory level $I_1(t)$ increases at a rate $(1-\theta)xP$ up to time t_1. At this time reworking process of imperfect items is started, therefore from time t_1 inventory level $I_1(t)$ decreases and reaches zero at time t_2 when reworking process is completed.

Differential equations governing inventory level $I(t)$ of perfect items at any time t in the time interval $[0,T]$ are as given as follows:

$$\frac{dI(t)}{dt}=(P-xP)-(a+bt), \qquad 0\le t\le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt}=P-(a+bt), \qquad t_1\le t\le t_2 \qquad (2)$$

$$\frac{dI(t)}{dt}=-(a+bt), \qquad t_2\le t\le T \qquad (3)$$

Using the boundary conditions $I(0)=0,\ I(T)=0$ the solution of the above equations are obtained as:

$$I(t)=(P-xP-a)t-\frac{bt^2}{2}, \qquad 0\le t\le t_1 \qquad (4)$$

$$I(t)=(P-a)t-\frac{bt^2}{2}-xPt_1, \qquad t_1\le t\le t_2 \qquad (5)$$

$$I(t)=-at-\frac{bt^2}{2}+Pt_2-xPt_1, \qquad t_2\le t\le T \qquad (6)$$

Quantity reworked is $(1-\theta)xPt_1=\int_{t_1}^{t_2}Pdt=Pt_2-Pt_1$, which gives

$$t_2=\{(1+(1-\theta)x\}t_1 \qquad (7)$$

Substituting value of t_2 from equation (7) in equation (6), the equation (6) changes to:

$$I(t) = -at - \frac{bt^2}{2} + (1-\theta x)Pt_1, \qquad t_2 \le t \le T \tag{8}$$

Using the boundary conditions $I(T) = 0$ in equation (8), an expression in T, is obtained as:

$$aT + \frac{bT^2}{2} = (1-\theta x)Pt_1$$

which gives $$T = -\frac{a}{b} + \left(\frac{a^2}{b^2} + \frac{2}{b}(1-\theta x)Pt_1\right)^{\frac{1}{2}} \tag{9}$$

Differential equation governing the inventory level $I(t_1)$ of imperfect inventory (to be reworked) in the interval $[0, t_2]$ are given as follows:

$$\frac{dI_1(t)}{dt} = (1-\theta)xp \qquad 0 \le t \le t_1 \tag{10}$$

$$\frac{dI_1(t)}{dt} = -P \qquad t_1 \le t \le t_2 \tag{11}$$

Using the boundary conditions $I_1(0)$ & $I_1(t_2) = 0,$ the solution of the above equations are obtained as:

$$I_1(t) = (1-\theta)xPt \qquad 0 \le t \le t_1 \tag{12}$$

$$I_1(t) = -Pt + Pt_2 \qquad t_1 \le t \le t_2 \tag{13}$$

Now, the following observations can be made about the model:

(1) Total production quantity Q in time $t_1 = Pt_1$ (14)

(2) Total defective items produced Q_d in time $t_1 = xPt_1$ (15)

(3) Total good items produced in time $t_1 = Q - Q_d$

(4) Total scrap quantity produced Q_s in time $t_1 = x\theta Pt_1$ (16)

(5) Total Rework quantity produced in time $t_1 = Q_d - Q_s = (1-\theta)xPt_1$ (17)

Total Cost consists the following terms:

(1) Setup Cost $= A$.

(2) Production Cost $P.C = C_p Q = C_p Pt_1$ (18)

(3) Reworking Cost $R.C = C_r(1-\theta)xPt_1$ (19)

(4) Disposal Cost $D.C = C_d \theta xPt_1$ (20)

(5) Screening Cost of defective items $Sr.C = C_q Pt_1$ (21)

(6) Perfect Inventory Holding Cost

$$P.H.C = C_h\Big[\int_0^{t_1}((P - xP - a)t - \frac{bt^2}{2})dt + \int_{t_1}^{t_2}((P-a)t - \frac{bt^2}{2} - xPt_1)dt$$
$$+ \int_{t_2}^{T}(-at - \frac{bt^2}{2} + (1-\theta x)Pt_1)dt \tag{22}$$

$$=C_h[(2\theta x^2+2\theta x-1-x-x^2-\theta^2x^2)\frac{Pt_1^{\,2}}{2}-\frac{aT^2}{2}-\frac{bT^3}{6}+(1-\theta x)Pt_1T] \tag{23}$$

(7) Imperfect Inventory Holding Cost

$$I.H.C=C_h[\int_0^{t_1}(1-\theta)xPtdt+\int_{t_1}^{t_2}(-Pt+Pt_2)dt]$$

$$=C_h[\frac{1}{2}((1-\theta)(1+(1-\theta)x)xPt_1^{\,2}] \tag{24}$$

Consequently the total Cost $TC(t_1)$ per unit time is a function of t_1 and is given by:

$$TC(t_1)=\frac{1}{T}[A\;+\;P.C\;+\;R.C\;+\;D.C\;+\;Sr.C\;+\;I.H.C\;+\;P.H.C]$$

$$=\frac{1}{T}\Big[A\;+\;C_pPt_1\;+\;C_r(1-\theta)xPt_1\;+\;C_d\theta xPt_1\;+\;C_qPt_1\;+C_h\{(2\theta x^2+2\theta x-1-x-x^2-\theta^2x^2)\frac{Pt_1^{\,2}}{2}$$

$$-\frac{aT^2}{2}-\frac{bT^3}{6}+(1-\theta x)Pt_1T\}+C_h\{\frac{1}{2}(1-\theta)(1+(1-\theta)x)xPt_1^{\,2}\}\Big]$$

$$=\frac{1}{T}\Big[A\;+\;(C_p+(1-\theta)xC_r\;+\;C_d\theta x\;+\;C_q)Pt_1\;+\frac{C_h}{2}(\theta x-1)Pt_1^{\,2}-\frac{C_h}{2}(aT^2+\frac{bT^3}{3})$$

$$+C_h(1-\theta x)Pt_1T\Big] \tag{25}$$

where T is a function of t_1 given by equation (9).

2.3 Solution Procedure:

According to the equation (25) $TC(t_1)$ is a function of t_1 . To minimize the total cost per unit time $TC(t_1)$ the optimal value of t_1 is obtained by solving the following equation:

$$\frac{\partial TC(t_1)}{\partial t_1}=0 \tag{26}$$

provided $\dfrac{\partial^2 TC(t_1)}{\partial t_1^{\,2}}>0.$

2.4 Numerical example 1:

Consider inventory system with the following parameters

$a=100, b=8, C_h=3, A=100, C_p=100, C_r=15, C_q=0.5, C_d=0.45, P=500, x=0.25, \theta=0.06$

Using the solution procedure described in the model the optimal results obtained are:

$t_1^*=3.42305, T^*=11.5357\; t_2^*=4.22747$

Optimal Production Quantity $Q^*=1711.53\dfrac{\delta y}{\delta x}$

Optimal Defective Quantity $Q_d^*=427.881$

Optimal Scrap Quantity $Q_s^*=25.6729$

Total Cost per Unit Time $TC(t_1^*)=13762.1$

Special cases:

Case 1: Defective items with no scrap i.e. $\theta = 0$:

In this case the total Cost $TC(t_1)$ is given by:

$$TC(t_1) = \frac{1}{T}[A + (C_p + xC_r + C_q)Pt_1 - \frac{C_h}{2}Pt_1^2 - \frac{C_h}{2}(aT^2 + \frac{bT^3}{3}) + C_h Pt_1] \quad (27)$$

$$\frac{\partial TC(t_1)}{\partial t_1} = \frac{1}{T}\frac{dT}{dt_1}\left[(A + (C_p + xC_r + C_q)Pt_1 - \frac{C_h}{2}Pt_1^2 - \frac{C_h}{2}(aT^2 + \frac{bT^3}{3}) + C_h Pt_1)\right]$$
$$-\left[(C_p + xC_r + C_q)P - C_h Pt_1 - C_h(aT + \frac{bT^2}{2})\frac{dT}{dt_1} + C_h + PT + C_h Pt_1\frac{dT}{dt_1}\right] = 0 \quad (28)$$

Where

$$T = -\frac{a}{b} + (\frac{a^2}{b^2} + \frac{2}{b}Pt_1)^{\frac{1}{2}} \quad (29)$$

$$\frac{dT}{dt_1} = \frac{P}{b}(\frac{a^2}{b^2} + \frac{2}{b}Pt_1)^{-\frac{1}{2}} \quad (30)$$

In an inventory system with the parameters of Numerical Example1 with $\theta = 0$ in place of $\theta = 0.06$ and applying the solution procedure described above the optimal results obtained are:

$t_1^* = 3.39498, T^* = 11.5961$

$t_2^* = 4.24372$

Optimal Production Quantity	$Q^* = 1697.49$
Optimal Defective Quantity	$Q_d^* = 424.373$
Total Cost per Unit Time	$T.C(t_1^*) =$ 13573.8 ‘

Case 2: When no defectives are produced i.e. $x = 0$:

In this case differential equations governing inventory level $I(t)$ changes to:

$$\frac{dI(t)}{dI} = P - (a + bt) \qquad 0 \le t \le t_1 \quad (31)$$

$$\frac{dI(t)}{dI} = -(a + bt) \qquad t_1 \le t \le T \quad (32)$$

The total Cost $T.C(t_1)$ is a function of t_1 and is given by:

$$T.C(t_1) = \frac{1}{T}\left[A + C_p Pt_1 - \frac{C_h}{2}Pt_1^2 - \frac{C_h}{2}(aT^2 + \frac{bT^3}{3}) + C_h Pt_1 T\right] \quad (33)$$

$$\frac{\partial T.C(t_1)}{\partial t_1} = \frac{1}{T}\frac{dT}{dt_1}\left[(A + C_p Pt_1 - \frac{C_h}{2}Pt_1^2 - \frac{C_h}{2}(aT^2 + \frac{bT^3}{3}) + C_h Pt_1)\right]$$
$$-\left[C_p P - C_h Pt_1 - C_h(aT + \frac{bT^2}{2})\frac{dT}{dt_1} + C_h + PT + C_h Pt_1\frac{dT}{dt_1}\right] = 0 \quad (34)$$

where T and $\frac{dT}{dt_1}$ are given by equations (29)and(30) respectively.

Numerical example 2:

In an inventory system with the parameters of Numerical Example1 with $x=0$ in place of x=0.25 and applying the solution procedure described above the optimal results obtained are:

$t_1^* = 3.48782, T^* = 11.8357,\ t_2^* = 3.48782$

Optimal Production Quantity $Q^* = 1743.91$

Total Cost per Unit Time $T.C(t_1^*) = 13019.3$

CONCLUSION

Differential equation formed in the model illustrates the behavior of the inventory at any instant of the time period of the model. Optimum total average cost the production inventory system is determined. Different special cases of the model (case of no defective items produced and case of no scrap produced) are discussed along with the models. Developed models in this paper may be helpful to some decision makers for certain type of imperfect production inventory system.

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