

## Algorithms Used for Reducing Peak-to-Average Power Ratio for OFDM Based Wireless Systems

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**Abstract**—Orthogonal frequency-division multiplexing (OFDM) effectively mitigates inter symbol interference (ISI) caused by the delay spread of wireless channels. Therefore, it has been used in many wireless systems and adopted by various standards. However, they are very sensitive to nonlinear effects due to the high peak-to-average power ratio (PAPR) owned by their transmitted signals. In this paper, we present a comprehensive survey on Peak-to-Average Power Reduction techniques for OFDM. We address basic OFDM as well as techniques to improve the performance of OFDM for wireless communications, including peak-to-average power ratio reduction.

**Keywords**—OFDM, PAPR, Precoding, Trigonometric Transforms.

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### I. INTRODUCTION

High data rate transmission over wireless channels is required by many applications. However, the symbol duration reduces with the increase of the data rate, and dispersive fading of the wireless channels will cause more severe inter symbol interference (ISI) if single carrier modulation, such as in time division multiple access (TDMA) or Global system for Mobile, is still in use. To reduce the effect of ISI, the symbol duration must be larger than the delay spread of wireless channels. OFDM is a combination of modulation and multiplexing. Multiplexing generally refers to independent signals, those produced by different sources. In OFDM [2-3] multiplexing is applied to independent signals but these independent signals are a sub-set of the one main signal. In OFDM signal first split into independent channels, modulated by data and then re-multiplexed to create the OFDM carrier. Due to the large number of sub carriers, OFDM systems have a large dynamic signal range with a very high peak-to-average power ratio (PAPR). As a result, the OFDM signal will be clipped when passed through a nonlinear power amplifier at the transmitter end. Clipping degrades the bit-error-rate (BER) performance and causes spectral spreading [4]. To achieve low peak-to-average power ratio several methods studied and compared. In this paper, we present a comprehensive survey of PAPR reduction techniques for OFDM. We start with basic OFDM system in Section II. In section III we address approaches to remove the amplitude limitations. Then, we address various PAPR reduction techniques in Section IV. Finally, we present our concluding remarks with comparison of various techniques we studied in Section V.

## **II. BASIC OFDM**

In this section, we will first describe basic OFDM and briefly explain performance parameters of OFDM for wireless communication, including channel estimation, time and frequency varying impairment mitigation.

### **2.1. OFDM**

To generate OFDM successfully the relationship between all the carriers must be carefully controlled to maintain the orthogonality of the carriers. For this reason, OFDM is generated by firstly choosing the spectrum required, based on the input data, and modulation scheme used. Each carrier to be produced is assigned some data to transmit. The required amplitude and phase of the carrier is then calculated based on the modulation scheme (typically differential BPSK, QPSK, or QAM). The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform. In most applications, an Inverse Fast Fourier Transform (IFFT) is used. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring the carrier signals produced are orthogonal.

The Fast Fourier Transform (FFT) transforms a cyclic time domain signal into its equivalent frequency spectrum. This is done by finding the equivalent waveform, generated by a sum of orthogonal sinusoidal components. The amplitude and phase of the sinusoidal components represent the frequency spectrum of the time domain signal. The IFFT performs the reverse process, transforming a spectrum (amplitude and phase of each component) into a time domain signal. An IFFT converts a number of complex data points, of length that is a power of 2, into the time domain signal of the same number of points. Each data point in frequency spectrum used for an FFT or IFFT is called a bin. The orthogonal carriers required for the OFDM signal can be easily generated by setting the amplitude and phase of each frequency bin, then performing the IFFT. Since each bin of an IFFT corresponds to the amplitude and phase of a set of orthogonal sinusoids, the reverse process guarantees that the carriers generated are orthogonal.

### **2.2. FFT points selection constraints in OFDM [1]**

As far as hardware implementation is concerned, the more the FFT points the more the processing power consumed that can be taken care of by the fast processors but in general more number of FFT points increases the resolution of the OFDM signal that gives better results, especially if the SNR is of the order of 12dB. The complexity of performing an FFT is dependent on the size of the FFT. This larger the FFT, the greater the number of calculations required. However, since the symbol period is longer, the increased processing required is less than the straight increase in processing to perform a single FFT.

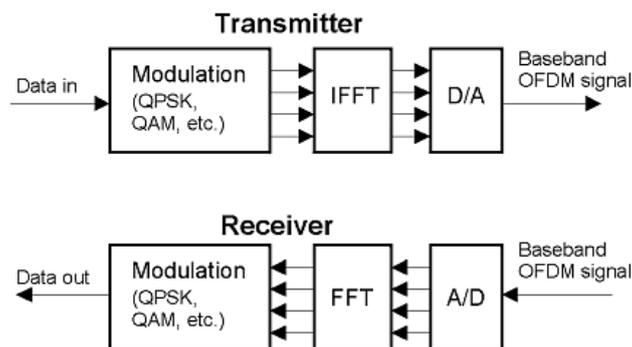
$$\text{Total number of calculations} = \frac{N \log_{10} N}{2 \log_{10} 2} + \frac{N \log_{10} N}{\log_{10} 2} \quad (1)$$

Hence total number of complex calculations required for a 32 point FFT is 240. The maximum time that can be taken in performing the calculation is once every symbol, thus once every symbol is 833 $\mu$ s. If we assume that the processor used requires two instructions to perform a single complex calculation, and that there is an overhead of 30% for scheduling of tasks and other processing, the minimum processing power required for this is then

$$\text{MIPS} = \frac{33792 * 2}{833 * 10^{-6}} * 1.3 * 10^{-6} = 105 \quad (2)$$

Thus the transmitter requires > 105 MIPS in order to implement to OFDM transmitter. The receiver will require just as much as the transmitter, thus a full OFDM transceiver will require >210 MIPS. This is a lot of processing required. Some chip DSPs are only 25-50 MIPS. Currently the fast general purpose DSP is produced by Texas Instruments which is capable of up to 1600MIPS which would make it plenty fast enough for an OFDM transceiver.

Fig.1. shows the configuration for a basic OFDM transmitter and receiver. The signal generated is at base-band and so to generate an RF signal the signal must be filtered and mixed to the desired transmission frequency. The sampled version of the baseband OFDM signal expressed as



**Figure 1. Basic OFDM Transmitter and Receiver**

$$S_n\left(m \frac{T_s}{N}\right) = \sum_{k=0}^{N-1} S_{n,k} e^{j2\pi k \Delta f m \frac{T_s}{N}} = \sum_{k=0}^{N-1} S_{n,k} e^{j \frac{2\pi m k}{N}} \quad (2)$$

$S_{n,k}$  be the complex symbols to be transmitted at the  $n$ th OFDM block, equation 1 is actually the inverse discrete Fourier transform (IDFT) of the transmitted symbols and can efficiently be calculated using FFT.

### 2.3.Channel Estimation

DDCE methods fit in systems operating in static or quasi-static channels. It particularly fits in systems in a slot transmission mode, such as wireless cellular systems [20]. PACE methods can reliably estimate channels both in static and time-varying channels by appropriately designing pilot patterns. When superimposed pilot sequence is considered, the bandwidth efficiency can be improved at the expense of an increase of transmit power.

### III. Approaches to Remove the Amplitude Limitations

First Approach is Inserting redundancy: Application of short block codes to enable a lower input back off in OFDM systems with four or eight subcarriers. Use of less number of subcarrier also improves spectral efficiency, gives twofold advantage. The principle of this idea is to select from the multitude of all possible OFDM blocks those which fulfill the condition  $s(t) < A_0$  at a given input back off. These suitable blocks meeting to the condition are assigned to different data bit sequences by a code. Beyond limiting the amplitude of the resulting OFDM signals, these codes can also be used for error correction at the receiver.

Second Approach is the same data sequence can be represented by several different OFDM blocks. The transmitter generates all possible signals corresponding to a data sequence and chooses the most suitable one for transmission. The receiver must additionally be told which of the signals has been chosen. This can be achieved with a little redundancy. If differential modulation is applied between adjacent subcarriers, the receiver does not even need any side information.

However, in this case, on several subcarriers, reference symbols are transmitted for the differential demodulation. This scheme allows us, for example to decrease the input back off from 12dB to 10dB at the same level of out-of-band interference.

Third approach is suggested in some literature that realizes an OFDM transmission with a constant envelope using 50% redundancy. In this scheme, instead of one OFDM block, two blocks are transmitted, calculated from  $s(t)$ . However, this calculation is non-linear and causes out-of-band interference. The objective of this approach is not to avoid out-of-band interference but to avoid interference of the OFDM signal.

#### IV. PEAK-TO-AVERAGE POWER REDUCTION TECHNIQUES

In fig. 2  $N$  is the baseband-modulated symbols per OFDM block is considered with Pre-coded Matrix  $P$  with dimensions  $N \times L$  is defined. Pre-coded symbols transmitted over the different subcarriers of the OFDM-modulation scheme. OFDM transmitted signal can be written as follows:

$$x(t) = \sum_{i=0}^{L-1} Y_i e^{j2\pi i \frac{t}{T}}, i = 0, 1, \dots, L-1 \quad (3)$$

Maximum PAPR obtained in [4] is written as follows

$$PAPR_{\max} = \frac{1}{N} \max_{0 \leq t < T} \left( \sum_{m=0}^{N-1} \left| \sum_{i=0}^{L-1} p_{i,m} e^{j2\pi i \frac{t}{T}} \right|^2 \right) \quad (4)$$

In (4) it is assumed that  $|X_m|^2 = E_s$ ,  $m=0, 1, 2, \dots, N$

The method proposed in [4] is data independent and avoids block based optimization and reduce PAPR in OFDM considerably. As BER performance is considered it is very similar to block coding and linear constellation pre-coding (LPC). Result of this method is compared with other methods and described in Section IV.

In [8] PAPR reduction scheme is proposed for coded OFDM system in which a convolutional channel code is deployed. Method is based on selective mapping (SLM) technique and no explicit side information is needed at the receiver side.

$$PAPR = \frac{\max_{0 \leq m \leq LN} |x_m|^2}{E\{|x_m|^2\}} \quad (5)$$

$x_m$  are the time domain signal samples and defined as follows :

$$x_m = x\left(\frac{mT}{L}\right) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi km}{LN}}, 0 \leq m < LN. \quad (6)$$

In [8] recursive convolutional code is used as scrambler and by inserting several dummy bits, it can generate sufficiently different candidate OFDM signals. In [9] analysis of the probability density functions of various types of precoded OFDM signals is done. In [9] it is showed that the precoders

can change the statistical properties of OFDM signals and hence the distribution of PAPR. Comparison with conventional is done and upto 4.5dB difference is achieved. In comparison with [4] , [9] used Walsh-Hadamard Transform (WHT) based precoder matrix. In [9] WHT matrix has dual role as it can based on Zadoff-Chu based and GCL based also. PAPR achieved using Zadoff Chu based OFDM is much less as compared to WHT-OFDM. 3B/4B coding scheme is proposed in [10] which can reduce PAPR significantly, and its complexity and effectiveness are totally independent of the number of OFDM sub-carriers. 3B/4B encoding is applied after the IFFT of the transmitter. At the receiver system uses error correction according to the 3B/4B encoding rule first to reduce the BER in the time domain first. Then it decodes the 3B/4B codeword before the FFT. Decimal numbers from 0 to 7 represented in 3B as from 000 to 111 and in 4B as from 0001 to 1111. Best thing in this method is that as we are going to increase the number of Sub carriers more reduction in PAPR is achieved. We can say that 3B/4B method is more immune against the increase of the sub carrier number and high BER performance. It also has low implementation complexity , which is also independent of the number of sub carriers N.

SLM is used to generate U different and statistically independent sequences of data which are named as no-0,1,1,...(U-1) . Then the sequence which has the lowest PAPR is selected for transmission. The advantage of this scheme is that no signal distortion is introduced. The disadvantage is that the receiver needs to know that which sequence was selected and transmitted. To overcome such type of problem technique based on BCH codes proposed in [11] . A linear block code C is a nonempty of n-tuples binary, called codeword , such that the sum of any two code words is a valid codeword. The code C is referred as an (n,k) code where n is the block length and k is the dimension of the code respectively.

$$n = 2^m - 1, m \geq 3, n - k \leq mt, t = \left\lceil \frac{d_{\min} - 1}{2} \right\rceil \quad (7)$$

For a minimum hamming distance of  $d_{\min}$  , the code can correct t errors/word. In [11] combination of (5,15) and (7,15) of BCH codes combined with another technique for PAPR reduction is used. Though this technique uses BCH codes but PAPR reduction is very less compared to 3B/4B coding. Very effective precoding technique with dual ability to both reduce PAPR and condition the OFDM signal against frequency selective fading channels is proposed in [16]. Method demonstrates that by advisedly crating diversity among the subcarriers of one OFDM symbol, and exploiting it with a dirty paper coding (DPC) approach , excellent BER performance is achieved without reducing transmission rate. Low APR factor can also be obtained through a proper selection of diversity matrix that distributes the power of each modulated symbol over the OFDM block. Over Selective Mapping (SLM), Selective vector perturbation (SVP) has additional benefit of reducing PAPR factor in OFDM systems. To prevent complexity technique uses multiple representations of V(i) (V is interference channel vector) are produced by cyclically redistributing the rows in V(1) . where  $V_{m,n}(i)$  described as follows

$$V_{m,n}^{(i)} = V_{m+i,n}^{(1)} \quad , i=1, \dots, P \quad (8)$$

By advisedly creating and selecting the diversity matrix and exploiting it with vector perturbation , great BER performance is obtained together with low PAPR factor. It is shown in [16] that the SVP technique outperforms coded OFDM under low SNR without causing data rate loss , and achieves slightly better PAPR reduction ability than conventional MAR techniques.

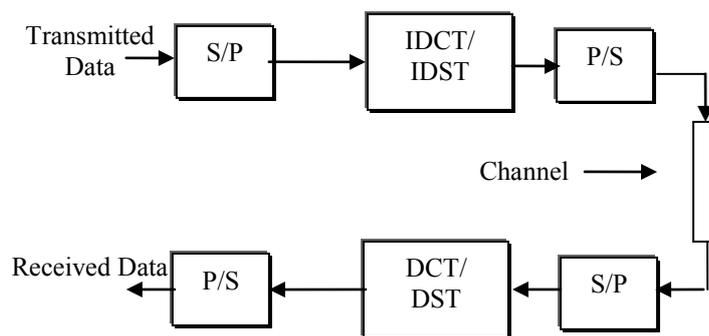
#### 4.1 TRIGONOMETRIC BASED OFDM SYSTEMS

Multicarrier modulation (MCM) techniques including OFDM and discrete multi tone (DMT) have been used extensively in wireless and wire line communication systems given their excellent performance in terms of data rate, spectral efficiency and tolerance to channel impairments at a reasonable implementation complexity. These multicarrier systems can be efficiently implemented using the FFT algorithm in order to build the DFT stage in the receiver and its IDFT in the transmitter. All the schemes to reduce PAPR described in II .A use IFFT and FFT blocks to modulate and demodulate the data symbols . Trigonometric transforms approach is described in [7] , such as discrete cosine transform (DCT) , discrete sine transform (DST) and discrete Hartley transform (DHT) along with their inverse .

A basic trigonometric based OFDM system is shown in Fig. 3. The IFFT/FFT blocks in the transmitter/receiver are replaced with IDCT/IDST or IDST/DST blocks. In [7] expressions for the complex envelopes of the trigonometric bases signals calculated using equation (8).

$$c_N(t) = \frac{2}{N} \sum_{k=0}^{N-1} A_k \cos/\sin\left(\frac{2\pi}{T}\left(k + \frac{1}{2}\right)t\right) \quad (9)$$

The complex envelop is presented with distributed random process with a single degree of freedom characterized with density function. In comparison with standard FFT based OFDM system less PAPR is achieved .



*Figure 3. Block diagram of basic trigonometric based OFDM system.*

PAPR expression is given below

$$P_N^{s/c} = \frac{\max_{0 \leq t \leq T} |s_N(t)|^2}{E\{|s_N(t)|^2\}} \quad (10)$$

#### V.

#### VI. COMPARISON OF VARIOUS TECHNIQUES AND CONCLUDING REMARKS

PAPR is a random variable and takes the values between zero and PAPRmax. A better measure of the PAPR of communication signals is then to consider the complementary cumulative distribution function defined as PPAPR=Pr(PAPR ≥ PAPR0). Where PAPR0 is the threshold. Fig.4. illustrates the complementary cumulative distribution function of the PAPR of various techniques described in this paper. It is observed that precoding schemes in [4] provides considerable gain as compared to other techniques in [5,7,8,9,11,12,15,16] for the case of N=512. For precoded schemes PAPR can be reduced 1.5dB with and overhead of 10%.

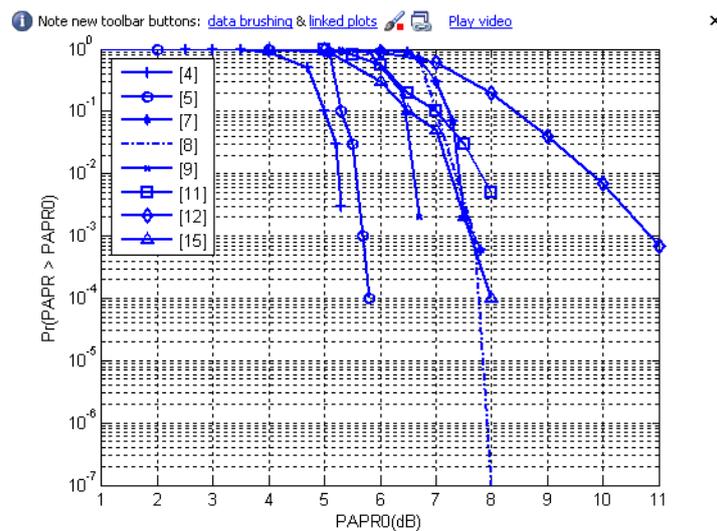


Fig. 4. Complementary distribution function of the PAPR for various methods studied.

As complementary cumulative distribution function only represents the distribution of the maximum amplitude within one OFDM signal but not the overall Mean Square Error (MSE) after the OFDM signal passed High Power Amplifier (HPA), it is no longer as one performance measure and therefore many approaches use BER performance with noisy channel to verify their schemes like [5,6, 8,10,13,14,16,18,19]. It is observed that scheme proposed in [5] provides considerable power gain for a bit-error probability of  $10^{-4}$ .

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