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A Green Supply Chain Inventory Model Considering the Short Life-Cycle Product

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Abstract— Due to the pressures of a short product life cycle and environmental sustainability, end-of life management is emerging as an important approach. Here we have modeled an integrated inventory control system with shortages under a short product life cycle, considering the synergy between the forward and the reverse logistics. We have considered that all the products in the reverse flow are collected, sorted and dissembled before being sent on to their next destinations. Thereafter, amount of the returned items is to be remanufactured and recycled. For the rest of the material, a salvaged option is also considered. Numerical verification of the theoretical results is provided. Finally the sensitivity analysis is reported.
Keywords- Supply chain inventory model, production, remanufacturing, recycling, Short life cycle Products.

I. INTRODUCTION

The increase of industrialization and globalization in developing countries creates more opportunities for manufacturing industry but concurrently increases environmental burden. Legislation and customer expectations increasingly force manufacturers to take back their products after use, which can be achieved through the reverse logistics. A reverse production system includes collection, sorting and remanufacturing processes for end-of-life products. Reverse distribution can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and the reverse channel. Schrady (1967) was the first who studied the reverse logistics assuming the traditional Economic Order Quantity (EOQ) model for repairable items. Nahmias and Rivera (1979) considered the model of Schrady (1967) for the case of finite repair rate and limited storage in the repair and production shops. Koh et al. (2002) generalized this model by assuming a limited repair capacity. Thereafter Alamri (2010), Singh and Saxena (2012), Singh and Saxena (2013) and Singh and Saxena (2014) have investigated the RL model with different assumptions.

The pressures of a short product life cycle and environmental sustainability make remanufacturing a reasonable choice. In recent years, Wee and Chung (2009) has developed a design life cycle with green components for the short life cycle product. Sometimes customers do not consider newly manufactured and remanufactured items as being interchangeable. There are a small number of researchers, who study the use of remanufacturing as a tool to serve secondary markets such as King et al. (2006), Jaber & El Saadany (2009), Konstantaras, Skouri, & Jaber (2010) and Hasanov et al. (2012).

In this paper we model a closed loop supply chain with two different quality standards; the first has a higher level of quality, while the second has a lower level of quality. Here the take back products are received by the supplier and subject to cleaning and dissembling, a ratio of material that conforms to certain quality standards is remanufactured, some items are recycled and rest of the used products are salvaged. Thereafter the newly manufactured material is delivered to the primary market, recycled products are treated as raw martial and the remanufactured items, acceptable in the secondary market, are sold at a cheaper rate.

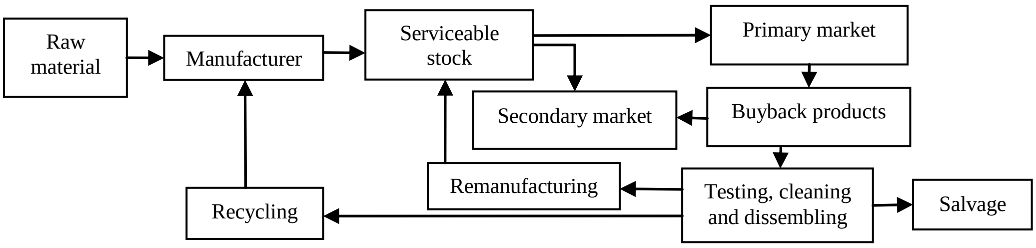

Fig. 1: Flow of inventory in the closed loop supply system

A general framework of the system is shown in Fig. 1. The next section, section 2 is for assumption and notations. Section 3 demonstrates the model development. Solution procedure to solve the optimization problem is given in Section 4. Section 5 shows a numerical example to illustrate the model and sensitivity analysis is also presented in this section. Concluding remarks are derived and future research topics are suggested in section 6.

II. NOTATIONS AND ASSUMPTIONS

P_m The production rate

D_m The demand rate associated with primary market

R_1 Returned rate associated with primary market

R Returned rate

h_r Unit holding cost per unit time associated with remanufactured items.

S_m Shortage cost associated with primary market.

LS_m Lost sale cost associated with primary market.

C_m Production cost

U_m Unit procurement cost

U_{R1} Unit acquisition cost associated with primary market

F_{rp} Fixed unit repairing cost

C_{cl} Variable cost including cleaning and disassembly cost during the collecting process

a_0 Fixed design-cost ratio for the green

P_r Remanufacturing rate

D_r The demand rate associated with secondary market

R_2 Returned rate associated with secondary market

h_R Unit holding cost per unit time associated with returned items.

h_m Unit holding cost per unit time associated with newly produced items.

S_r Shortage cost associated with secondary market.

LS_r Lost sale cost associated with secondary market.

C_{rp} Variable repair cost

S_{av} Salvage for the unusable items after cleaning, disassembly and sorting.

U_{R2} Unit acquisition cost associated with secondary market

F_{cl} Fixed cost including cleaning and disassembly cost during the collecting process

C_{sgn} Design cost for the green design

b_0 Variable design-cost ratio for the green

	design.
r_j	Reliability of the sub function j
M	Number of the life cycles before the component is recycled or disposed of.
F_r	Fixed cost during the remanufacturing process.
δ_r	The arrival rate of the components which can be remanufactured.
α	proportion of a batch of used items consumed in the recycling segment
η_r	proportion of D_r that is backordered
	design.
γ	Scaling parameter, salvage formulation
Y	Component-design-life cost.
C_r	Variable cost during the remanufacturing process.
δ_{rp}	The arrival rate of the components which can be repaired.
β	proportion of a batch of used items consumed in the remanufacturing segment
η_m	proportion of D_m that is backordered

III. MATHEMATICAL MODELLING AND ANALYSIS

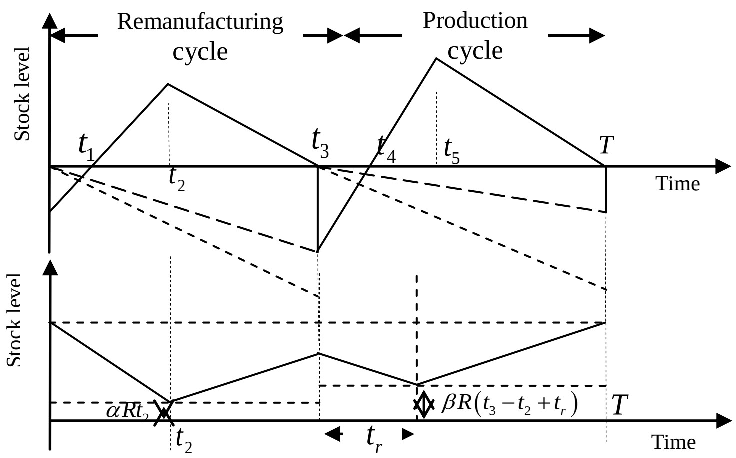

Fig. 2: Inventory variation of an EPQ model for green supply chain system.

Now, the per cycle cost components for the given unified inventory system are as follows:

Holding cost

$$= \frac{h_r}{2}\left\{(P_r - D_r)(t_2 - t_1)^2 + D_r(t_3 - t_2)^2\right\} + \frac{h_m}{2}\left\{(P_m - D_m)(t_5 - t_4)^2 + D_m(T - t_5)^2\right\} + h_R\left\{t_2^2\frac{P_r}{2} + t_r^2\frac{P_m}{2} + (\alpha+\beta)R\frac{T^2}{2} - RT(\alpha t_r + \beta t_2)\right\}$$

Shortage and Lost sale cost

$$= S_r\left\{(P_r - D_r)\frac{t_1^2}{2} + \eta_r D_r\frac{(T - t_3)^2}{2}\right\} + S_m\left\{\eta_m D_m\frac{t_3^2}{2} + (P_m - D_m)\frac{(t_4 - t_3)^2}{2}\right\} + LS_r(1-\eta_r)D_r(T - t_3) + LS_m(1-\eta_m)D_m t_3$$

Procurement, accusation, production, remanufacturing and repairing cost

$$= U_m P_m(t_5 - t_3 - t_r) + (U_{R1}R_1 + U_{R2}R_2)T + C_m P_m(t_5 - t_3) + \left\{\frac{F_r}{M} + MC_r P_r t_2\left[1 - e^{-\delta_r T}\right]\right\} + \left\{\frac{F_{rp}}{M} + MC_{rp}\alpha TR\left[1 - e^{-\delta_{rp} T}\right]\right\}$$

Cleaning, Sorting and Dissembling cost $= \{F_{cl} + C_{cl}RT\}$

Component design-life cost

$$Y(M,T) = C_{\text{sgn}}\left\{\frac{a_0}{M} + Mb_0\prod_{j=1}^{2} r_j\right\}$$

Salvage Cost $= S_{av}\gamma RT = S_{av}(1 - \alpha - \beta)RT$

Thus, the total cost per unit time of the underlying unified inventory system during the cycle is given by

$$TC(t_r,t_1,t_2,t_3,t_4,t_5,T)=\frac{1}{T}\Bigg[+C_mP_m(t_5-t_3)+U_mP_m(t_5-t_3-t_r)+(U_{R1}R_1+U_{R2}R_2)T+(F_{cl}+C_{cl}RT)+C_{\text{sgn}}\left\{\frac{a_0}{M}+Mb_0\prod_{j=1}^{2}r_j\right\}$$
$$+\left\{\frac{F_r}{M}+MC_rP_rt_2\left[1-e^{-\delta_rT}\right]\right\}+\left\{\frac{F_{rp}}{M}+MC_{rp}\alpha TR\left[1-e^{-\delta_{rp}T}\right]\right\}+\frac{h_r}{2}\left\{(P_r-D_r)(t_2-t_1)^2+D_r(t_3-t_2)^2\right\}+\frac{h_m}{2}\left\{(P_m-D_m)(t_5-t_4)^2\right.$$
$$\left.+D_m(T-t_5)^2\right\}+h_R\left\{t_2^2\frac{P_r}{2}+t_r^2\frac{P_m}{2}+(\alpha+\beta)R\frac{T^2}{2}-RT(\alpha t_r+\beta t_2)\right\}+S_r\left\{(P_r-D_r)\frac{t_1^2}{2}+\eta_rD_r\frac{(T-t_3)^2}{2}\right\}$$
$$+S_m\left\{\eta_mD_m\frac{t_3^2}{2}+(P_m-D_m)\frac{(t_4-t_3)^2}{2}\right\}+LS_r(1-\eta_r)D_r(T-t_3)+LS_m(1-\eta_m)D_mt_3-S_{av}(1-\alpha-\beta)RT\Bigg] \qquad (1a)$$

Here we have a cost function of the system in terms of t_1,t_2,t_3,t_4,t_5,t_r and T. Now the problem is to minimize TC but we have some relation between the variables as follows.

$P_rt_2=\beta RT$, $P_mt_r=\alpha TR$, $\gamma=(1-\alpha-\beta)$, $(P_r-D_r)(t_2-t_1)=D_r(t_3-t_2)$, $(P_m-D_m)(t_5-t_4)=D_m(T-t_5)$,

$\eta_mD_mt_3=(P_m-D_m)(t_4-t_3)$ and $\eta_rD_r(T-t_3)=(P_r-D_r)t_1$

From these relations the value of t_1,t_2,t_3,t_4,t_5 and t_r can be determined as a function of T

Where the value of t_1,t_2,t_3,t_4,t_5 and t_r are given as

$$t_1=\frac{T(D_r-R\beta)\eta_r}{(P_r-D_r)(1-\eta_r)},\ t_2=\frac{\beta RT}{P_r},\ t_3=\frac{T(R\beta-D_r\eta_r)}{D_r(1-\eta_r)},\ t_4=\frac{T\{P_m-D_m(1-\eta_m)\}(R\beta-D_r\eta_r)}{D_r(1-\eta_r)(P_m-D_m)} \text{ and}$$
$$t_5=\frac{T\left(P_mR\beta+D_m\left(D_r-R\beta(1-\eta_m)\right)\right)+D_rT(P_m+D_m\eta_m)\eta_r}{P_mD_r(1-\eta_r)}$$

Therefore the total variable cost function will be the function of T and M. For any given positive integer M the cost function (1) can be rewritten as

$$TC_M(T)=\frac{a}{T}+b+cT \qquad (2)$$

Where

$$a=\left\{F_{cl}+\frac{F_r}{M}+\frac{F_{rp}}{M}+c_{\text{sgn}}\left(\frac{a_0}{M}+Mb_0\prod_{j=1}^{2}r_j\right)\right\}$$

$$b=\Bigg\{C_{cl}R+(U_{R1}R_1+U_{R2}R_2)+S_{av}(1-\alpha-\beta)R+LS_r(D_r-R\beta)+\frac{LS_m(1-\eta_m)(R\beta-\eta_rD_r)D_m}{D_r(1-\eta_r)}-\frac{C_mD_m\{R\beta(1-\eta_m)-D_r(1-\eta_m\eta_r)\}}{D_r(1-\eta_r)}$$
$$-\frac{U_m\left[D_rR\alpha(1-\eta_r)+D_m\{R\beta(1-\eta_m)-D_r(1-\eta_m\eta_r)\}\right]}{D_r(1-\eta_r)}\Bigg\}$$

$$c=\Bigg\{h_rR\left(P_r\alpha(P_m-R\alpha)+P_mP_r\beta-P_mR\beta^2\right)+\frac{S_r(D_r-R\beta)^2(P_r-D_r(1-\eta_r))\eta_r}{2D_r(P_r-D_r)(1-\eta_r)^2}+\frac{D_mS_m(R\beta-\eta_rD_r)^2(P_m-D_m(1-\eta_m))\eta_m}{2D_r^2(P_m-D_m)(1-\eta_r)^2}$$
$$+C_rMR\beta\delta_r+C_{rp}MR\alpha\delta_{rp}+\frac{h_r\left(P_rR\beta+D_r\left(R\beta(1-\eta_r)+P_r\eta_r\right)\right)^2}{2D_r(P_r-D_r)P_r(1-\eta_r)^2}+\frac{D_mh_m\left(P_m(D_r-R\beta)+D_m\{R\beta(1-\eta_m)-D_r(1-\eta_m\eta_r)\}\right)^2}{2D_r^2(P_m-D_m)P_m(1-\eta_r)^2}\Bigg\}$$

The purpose of this study is to derive the optimal number of deliveries and the replenishment cycle time by determining the optimal values of M and T that maximize the total profit.

Taking the first order partial derivative of the profit function TC w. r. t. T, equating to zero and solving it, we obtain the optimal value of T at T^*

$T^{*}=\left(\frac{a}{c}\right)^{1/2}$, where the values of a and c is given above.

Since it is impossible to have number of shipment in decimal value, the value of M has to be rounded using the inequalities $TC\left(\left(M-1\right)|\,\mathrm{T}\right)\geq TC\left(M|\,\mathrm{T}\right)\leq TC\left(\left(M+1\right)|\,\mathrm{T}\right)$

Proposition 1. The total cost per unit time TC has a unique global optimal solution if $G>0$ where

$$G=\frac{\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)\left\{3\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)+4F_{cl}M\right\}+6b_0c_{\text{sgn}}\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)M^2r_1r_2-b_0^2c_{\text{sgn}}^2M^4r_1^2r_2^2}{M^4T^4}$$

Proof. For the total cost TC the Hessian matrix is given below:

$$H=\begin{pmatrix}\frac{\partial^2TC}{\partial T^2} & \frac{\partial^2TC}{\partial T\partial M}\\ \frac{\partial^2TC}{\partial M\partial T} & \frac{\partial^2TC}{\partial M^2}\end{pmatrix}=\begin{pmatrix}\frac{2\left\{F_{cl}+\frac{F_r}{M}+\frac{F_{rp}}{M}+c_{\text{sgn}}\left(\frac{a_0}{M}+Mb_0\prod_{j=1}^{2}r_j\right)\right\}}{T^3} & \frac{\left\{F_r+F_{rp}+c_{\text{sgn}}\left(a_0-b_0r_1r_2M^2\right)\right\}}{M^2T^2}\\ \frac{\left\{F_r+F_{rp}+c_{\text{sgn}}\left(a_0-b_0R_1R_2M^2\right)\right\}}{M^2T^2} & \frac{2\left\{F_r+F_{rp}+c_{\text{sgn}}a_0\right\}}{M^3T}\end{pmatrix}$$

$$=\frac{\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)\left\{3\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)+4F_{cl}M\right\}+6b_0c_{\text{sgn}}\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)M^2r_1r_2-b_0^2c_{\text{sgn}}^2M^4r_1^2r_2^2}{M^4T^4}$$

If

$$\frac{\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)\left\{3\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)+4F_{cl}M\right\}+6b_0c_{\text{sgn}}\left(F_r+F_{rp}+a_0c_{\text{sgn}}\right)M^2r_1r_2-b_0^2c_{\text{sgn}}^2M^4r_1^2r_2^2}{M^4T^4}>0$$

The Hessian is positive definite

Hence the cost function of the system TC is jointly convex function of M *and* T and the stationary point for our optimization problem exists and is also unique.

IV. NUMERICAL ANALYSIS

Example 1. We have considered the following input parameters in appropriate units.

$P_m=8000, P_r=6000, D_m=6000, D_r=2500, U_{R1}=8, U_{R2}=5, C_{\text{sgn}}=500,\ a_0=8, b_0=1, r_1=0.999, r_2=0.98, U_m=25,$
$C_m=100, F_{cl}=1000, C_{cl}=2, F_r=5000, F_{rp}=4000, C_r=25, C_{rp}=10, \delta_r=0.002, \delta_{rp}=0.002, R_1=1500, R_2=625,$
$LS_m=150, LS_r=60, S_m=100, S_r=45, \alpha=0.2, \beta=0.6, \eta_m=0.2, \eta_r=0.2, S_{av}=25, h_R=10, h_m=70, h_r=30,$

Applying the solution procedure given in the last section we derive the optimal solution and results are presented in the Table 1. While the convexity of the reverse logistics inventory model is shown in Figure 3.

Table2. The optimal results for the inventory model under the above parametric values as in example 1.

M	t_r	t_1	t_2	t_3	t_4	t_5	T	TC
5	0.0217192	0.0357728	0.0868767	0.158422	0.253475	0.369992	0.408831	899835

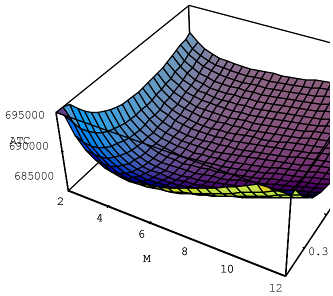

Fig. 3 Convexity of the reverse logistics inventory model is derived

Example 2. To study the effects of the parameter changes on the optimal solutions derived by the proposed method, this investigation performs a sensitivity analysis by increasing or decreasing the parameters, one at a time.

		M	t_r	t_1	t_2	t_3	t_4	t_5	T	TC
P_m	7200	5	0.02490	0.03692	0.08966	0.16351	0.32702	0.40614	0.42196	898913.
	7600	5	0.02340	0.03662	0.08893	0.16217	0.28380	0.39015	0.41851	899150.
	8000	5	0.02171	0.03577	0.08687	0.15842	0.25347	0.36999	0.40883	899835
	8400	5	0.02012	0.03480	0.08452	0.15412	0.23119	0.35016	0.39774	900659.
	8800	5	0.01868	0.03386	0.08223	0.14995	0.21422	0.33201	0.38698	901505.
P_r	5400	5	0.02183	0.04341	0.09705	0.15929	0.25486	0.37202	0.41107	899673.
	5700	5	0.02177	0.03923	0.09169	0.15884	0.25414	0.37097	0.40991	899756.
	6000	5	0.02171	0.03577	0.08687	0.15842	0.25347	0.36999	0.40883	899835
	6300	5	0.02166	0.03286	0.08253	0.15803	0.25285	0.36908	0.40782	899908.
	6600	5	0.02161	0.03039	0.07860	0.15767	0.25227	0.36823	0.40689	899976.
D_m	5400	5	0.02114	0.03482	0.08456	0.15420	0.21826	0.33955	0.39795	820993.
	5700	5	0.02144	0.03531	0.08576	0.15639	0.23391	0.35481	0.40360	860393.
	6000	5	0.02171	0.03577	0.08687	0.15842	0.25347	0.36999	0.40883	899835
	6300	5	0.02189	0.03605	0.08756	0.15967	0.27802	0.38358	0.41207	939427.
	6600	5	0.02180	0.03590	0.08720	0.15902	0.30895	0.39263	0.41038	979373.
D_r	2250	5	0.02122	0.02597	0.08490	0.18312	0.29299	0.37290	0.39954	894023.
	2375	5	0.02158	0.03082	0.08635	0.17110	0.27377	0.37322	0.40638	896539.
	2500	5	0.02171	0.03577	0.08687	0.15842	0.25347	0.36999	0.40883	899835
	2625	5	0.02165	0.04076	0.08663	0.14559	0.23295	0.36399	0.40767	903775.
	2750	5	0.02144	0.04580	0.08579	0.13305	0.21288	0.35603	0.40374	908253.

Observations

1. From the table it is observed that the remanufacturing and production cycle time period is positively sensitive to the change in the parameter D_m while negatively sensitive to the change in P_m, P_r, D_r
2. It is noted that total cost is increasing as the demand, production and remanufacturing increasing.

CONCLUSION

This paper discusses forward/reverse logistics network design by considering short product life-cycle. An integrated inventory model is developed with reverse flow of material including collection, production, recycling, remanufacturing, salvage centre, primary and secondary market. The theoretical results are illustrated through the numerical examples. Further, the effects of different parameters are compared. This model can provide an efficient opportunity for managers to make proper decisions for designing logistics network among various facilities with various parameters. For future research the model can be expanded to include the elements of reliability involved in the reverse logistics network design problem. Economies of scale and learning effects too will have their impact.

REFERENCES

[1] A. Alamri, Theory and methodology on the global optimal solution to a general Reverse logistics Inventory Model for deteriorating items and time-varying rates: Computer & Industrial Engineering, vol. 60, pp. 236-247, 2010.

[2] P. Hasanov, M. Y. Jaber and S. Zolfaghari, Production, remanufacturing and waste disposal models for the cases of pure and partial backordering: Applied Mathematical Modelling, vol. 36, pp. 5249–5261, 2012.

[3] M. Y. Jaber and A. M. A. El Saadany, The production, remanufacture and waste disposal model with lost sales. International Journal of Production Economics, vol. 120(1), pp. 115–124, 2009.

M. King, S. C. Burgess, W. Ijomah, and C. A. McMahon, Reducing waste: Repair, recondition, remanufacture or recycle? Sustainable Development, vol. 14(4), pp. 257–267, 2006.

[4] S. G. Koh, H. Hwang, K. I. Sohn & C. S. Ko, An optimal ordering and recovery policy for reusable items. Computers and Industrial Engineering, vol. 43(1–2), pp. 59–73, 2002.

[5] Konstantaras, K. Skouri, and M. Y. Jaber, Lot sizing for a recoverable product with inspection and sorting, Computers & Industrial Engineering, vol. 58 (3), pp. 452–462, 2010.

[6] S. Nahmias & H. Rivera, A deterministic model for a repairable item inventory system with a finite repair rate. International Journal of Production Research, vol. 17(3), pp. 215–221, 1979.

[7] D. A. Schrady, A deterministic Inventory model for repairable items. Naval Research Logistics Quarterly, vol. 14(3), pp. 391–398, 1967.

[8] S. R. Singh and N. Saxena, A Closed Loop Supply Chain System with Flexible Manufacturing and Reverse Logistics Operation under Shortages for Deteriorating Items. Procedia Technology, Vol. 10, pp. 330 – 339, 2013.

[9] S. R. Singh and N. Saxena, An Integrated Inventory Model with Remanufacturing of Secondary Material under Shortages, Proceedings of 3rd International Conference on Recent Trends in Engineering & Technology, pp. 354-359, 2014.

[10] S. R. Singh and N. Saxena, An Optimal Returned Policy for a Reverse Logistics Inventory Model with Backorders, Advances in Decision Sciences, vol. 2012, Article ID 386598, 21 pages, 2012.

[11] H. M. Wee and C. J. Chung, Optimising replenishment policy for an integrated production inventory deteriorating model considering green component-value design and remanufacturing. International Journal of Production Research, vol. 47(5), pp. 1343 -1368, 2009.

