

STATE FEEDBACK CONTROL OF PMLSM DRIVE BY USING THE SVPWM TECHNIQUE

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Abstract– In this paper the optimal state feedback control design of permanent magnet linear synchronous motor (PMLSM) is proposed. The mathematical model of PMLSM is obtained and linearised by introducing a variable in to the model which will decouple the linear and non linear components which is used to get the linearised model. The discrete linear quadratic regulator is used to design the state feedback control by finding the optimal gain matrix. The linearised PMLSM equations are used to form the objective function by incorporating weighting matrices Q and R. The motor is controlled by conventional PI method and then by the proposed method. In each method the system behavior is studied and the results are compared to evaluate them.

Keywords: Linear Quadratic Regulator, decoupling unit, permanent magnet linear synchronous motor (PMLSM)

I. INTRODUCTION

Permanent Magnet Linear Synchronous Motors produce straight line motion unlike rotary motors which move in rotational motion [1]. Linear machines are not so different from rotary ones. When the rotary motor is cut radially and made it flat then it will function as linear motor. The PMLSM has become significant and affordable due to the recent progress in power electronic devices and the reduction in cost of rare earth metals used in the manufacturing of permanent magnets. These PMLSMs are replacing the conventional rotary motors where the linear motion is required such as in lifts, conveyor belts and railway traction etc as there is no backlash error in linear motors. The absence of mechanical couplings, brushes, gears and belt mechanisms in PMLSM enables to get higher efficiency and precision [2]. These motors can be used in highly polluted areas such as shipyards mines etc. It is also used in ambience in which humans cannot work. Unlike rotary motors PMLSM speed does not depend on number of poles and the mover moves in synchronism with the travelling magnetic field of the forcer [3]. The mathematical model of PMLSM is similar to the permanent magnet synchronous motor (PMSM) and is derived by using the analogy between them. The motor is controlled by state feedback control method in which the outputs are measured and feedback to generate the control signals. Since the variables, considered to design the controller are internal to the system, a quality control can be obtained. The optimal gain matrix is to be properly selected in order to design an efficient state feedback control system.

Classical control methods can be successfully applied to single input single output (SISO) but they will become cumbersome for multi input multi output systems (MIMO). Hence optimal control methods are employed to analyze and to obtain the desired performance objectives where as it is not possible with conventional methods like pole placement methods. The optimal control infers the formulation of best control system possible for any desired performance objective. In the numerous ways available to realize a performance objective, there will be one way through which it can be

achieved with minimum amount of energy which is obtained by optimal control. Linear control is a special category of optimal control where the convergence of optimal solution is certain. Linear Quadratic Regulator is an optimal control method which directly addresses the performance objectives by minimizing the input energy [4]. The total energy of system comprises the transient energy and control energy. An objective function is formed by integrating this total energy with respect to time. This objective function is minimized by using discrete Linear Quadratic Regulator through which the optimum state feedback gain matrix is obtained. Since the LQR is only applied to linear systems the non linear model of PMLSM is linearised by introducing a new variable into the equations there by decoupling the nonlinear and linear terms [5]. The input voltage is modified in such a way that it nullifies the non linear terms in the equations. The non linear voltage component is added to the linear control voltage generated by state feedback control law and the resultant voltage is given to the drive.

The space vector pulse width modulation technique is used to control the voltage source inverter which is connected to the PMLSM [6]. The drive is simulated in both conventional PI control method and proposed LQR method and The simulation results are compared to study the performance of both methods and to distinguish them. The discrete LQR is tuned by considering various weighting matrices i.e. Q and R to obtain various performance characteristics. These results are evaluated to obtain the best control method.

II. MODELLING OF PMLSM

The mathematical modeling is performed in rotary reference frame since it will eliminate the time varying inductances and reduce the complexity in analyzing the transient and steady state analysis of the drive system. At first the three phase should be transformed into two phase system i.e. d-q axis rotating with the rotor speed since the induced emf follows the rotor position. The mathematical modeling obtained as

$$\frac{di_q}{dt} = -\frac{R_s}{L_s}i_q + \frac{v_q}{L_s} - \frac{\pi v_e}{\tau L_s}\psi_{pm} - \frac{\pi}{\tau}v_e i_d \quad (1)$$

$$\frac{di_d}{dt} = -\frac{R_s}{L_s}i_d + \frac{v_d}{L_s} + \frac{\pi}{\tau}v_e i_q \quad (2)$$

$$\frac{dv_r}{dt} = \frac{1}{M}(F_e - F_L - Bv_e) = \frac{1}{M}(k_f i_q - F_L - Bv_e) \quad (3)$$

Whereas v_e represents linear velocity M is mass and F is linear force B is coefficient of friction.

III. PMLSM LINEARISED MODEL

The linearised model of PMLSM can be obtained by using equation (1) & (2). The concept of decoupling is used in linearization of PMLSM. The nonlinearities are occurred due to the cross couplings of the speed term v_e with current terms i.e. the rotational component of voltage in the rotor reference frame. In the decoupling method a new variable is introduced so that it will eliminate the non linearity in the system. The input voltage is modified in such way that it will cancel the nonlinearity and for that the equivalent voltages have to be found out for which the nonlinearity becomes zero. Consider the equation (1) and make the non linear terms equal to zero.

$$\frac{v_q}{L_s} - \frac{\pi v_e}{\tau L_s} \psi_{pm} - \frac{\pi}{\tau} v_e i_d = 0 \quad (4)$$

$$v_{qe} = \frac{\pi}{\tau} (v_e \psi_{pm} + v_e i_d L_s) \quad (5)$$

Consider the equation (2) and make the non linear terms equal to zero

$$\frac{v_d}{L_s} + \frac{\pi}{\tau} v_e i_q = 0 \quad (6)$$

$$v_{de} = -\frac{\pi}{\tau} (v_e i_q L_s) \quad (7)$$

The equivalent d-q voltages are obtained in the equation (5) & (7) and these terms are substituted in the equation (1) & (2) respectively as follows

$$\frac{di_q}{dt} = -\frac{R_s}{L_s} i_q + \frac{(v_q + v_{qe} - v_{qe})}{L_s} - \frac{\pi v_e}{\tau L_s} \psi_{pm} - \frac{\pi}{\tau} v_e i_d \quad (8)$$

$$\frac{di_d}{dt} = -\frac{R_s}{L_s} i_d + \frac{(v_d + v_{de} - v_{de})}{L_s} + \frac{\pi}{\tau} v_e i_q \quad (9)$$

Hence the linear voltages are v_{ql} and v_{dl} where

$$v_{ql} = v_q - v_{qe} \quad \text{and} \quad v_{dl} = v_d - v_{de}$$

The linearised model of PMLSM is

$$\begin{bmatrix} \dot{i}_q \\ \dot{i}_d \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 \\ 0 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \end{bmatrix} \begin{bmatrix} v_{ql} \\ v_{dl} \end{bmatrix} \quad (10)$$

The nonlinear component of voltage which is eliminated will be added to the control voltages generated from the linear model and given to the inverter.

IV. LQR CONTROLLED PMLSM DRIVE

The block diagram of PMLSM drive with SVPWM and discrete linear quadratic drive is shown Fig.1. The state feedback control of the drive is obtained by the linearised model of the permanent magnet linear synchronous motor.

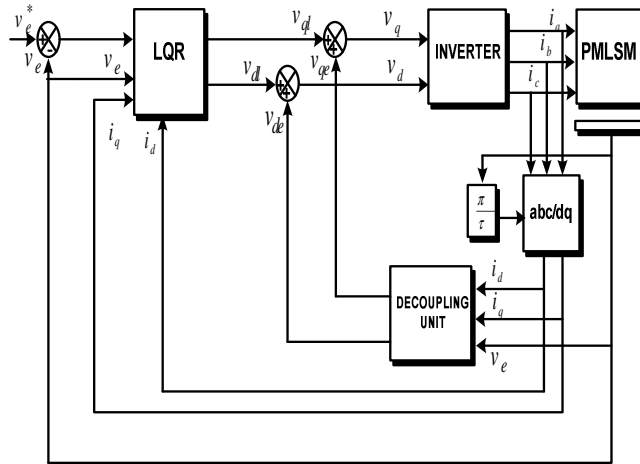


Fig. 1 the LQR controlled PMLSM drive

In the above diagram the PMLSM is controlled by state feedback control technique in which the output state variables are feedback to the controller through state feedback gain. Due to the coupled, non-linear model of the PMLSM, the use of the linear control theory to design a state feedback discrete speed controller is not possible. To accomplish this task the linearised mathematical model of PMLSM is considered. The design of a state feedback PMLSM speed controller requires the appointment of gain matrices. Techniques such as the LQR method have been used to achieve the designed behavior. Hence the equations are linearised by decoupling the non linear terms and the linear model is used to find the state feedback gain matrix and to design the Controller. Before these inputs are given to the machine the decoupled non linear terms are added and fed to the SVPWM inverter.

V. IMPLEMENTATION OF DISCRETE LQR

In order to design the discrete linear quadratic controller the linearised PMLSM model is considered and it is represented in the form of state space model

$$\dot{x} = Ax + Bu$$

The discrete model is obtained as

$$x(k+1) = Ax(k) + Bu(k) \quad (11)$$

As step signal is considered as reference in order to get the zero steady state error, type one system should be designed. For the type one system the basic principle of operation needs an extra state variable which is the integral of the difference between the reference value and the measured value. v_{ref} is reference speed.

$$\dot{x}_{ni} = A_{ni}x_{ni} + B_{ni}u_{ni} + E_{ni}v_{ref} \quad (12)$$

From the Eq (10) we can observe that the state model is augmented by an internal model of step input (by adding the state variable e_c . This state variable is the integral of speed error.

$$e(t) = r \int (v_e(\tau) - v_{ref}(\tau)) d\tau \quad (13)$$

$$r = \frac{\pi}{\tau}$$

The control signals can be obtained by multiplying the state variables with the optimum state feedback gains found by Linear Quadratic Regulator method. From the equation (12) the state space model can be formed by considering the linearised mathematical model of PMLSM, the input voltages corresponding to the linear model and the error signal is also considered as a state variable to get zero steady state error.

$$\begin{bmatrix} \dot{i}_q \\ \dot{i}_d \\ \dot{v}_e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 & 0 \\ \frac{k_f}{M} & 0 & -\frac{B}{M} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ v_e \\ e \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{dl} \\ v_{ql} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} v_{ref}$$

The state feedback variables are augmented with the gains to get the control signal as shown in the Fig. 2 and the control law is given in equation (13)

$$u_{ni}(t) = k_e e(t) - k_x x_n(t) \tag{14}$$

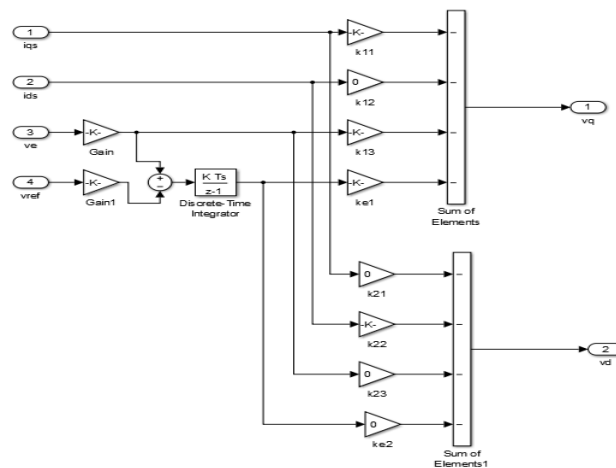


Fig 2 circuit diagram of controller

The optimum state feedback gain matrix obtained as k

$$k = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{e1} \\ k_{21} & k_{22} & k_{23} & k_{e2} \end{bmatrix}$$

The discrete linear quadratic regulator will get the optimal gain value by considering the state feedback law through which the cost function is minimized.

$$u(n) = -kx(n)$$

The discrete cost function to be minimized is similar to the continuous cost function shown in the equation (15) & (16)

$$J = \int_t^{t_f} [U^T(\tau)R(\tau)U(\tau) + X^T(\tau)Q(\tau)X(\tau)]d\tau \tag{15}$$

The discrete objective function

$$J = \sum [U^T R U + X^T Q X] \tag{16}$$

The feedback gain matrix is obtained

$$k = \begin{bmatrix} 1.1263 & 0 & 3.0765 & 2.6708 \\ 0 & 1.1394 & 0 & 0 \end{bmatrix} \tag{17}$$

In the equation (16) the objective function comprises the input energy term $U^T R U$ and the transient term $X^T Q X$ where Q is the state weighting matrix and R is input weighting matrix. The Q matrix consists of the weights corresponding to the covariance of the individual state variables thus facilitating the selection of weights independently with respect to each other. By minimizing the minimizing the objective function the gain values are found and hence it infers that the motor is controlled with the lowest possible energy. The outputs of the controller are voltages of the linearised model and they are added to the non linear voltage components which are generated by using the decoupling unit and given to space vector pulse width modulated inverter as shown in the Fig.3. The SVPWM technique generates the reference space vector from the d-q voltages obtained from the controller. The looks up tables are predefined to the corresponding magnitude and phase of rotating space vector.

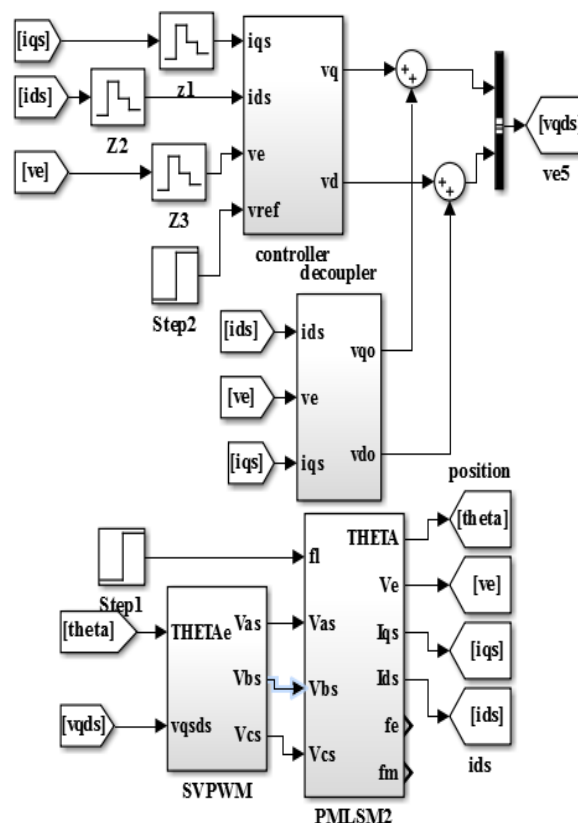


Fig 3 the circuit diagram of PMLSM drive

By considering the rotating space vector and the lookup tables the pulses are generated and given to the inverter switches to get the three phase output which will be given to the motor. By using the SVPWM technique the harmonics can be reduced and the switching losses are reduced. The SVPWM inverter will achieve 15% more efficiency than SPWM inverter. In this paper the PMLSM drive is operated with conventional pi method and with the proposed discrete LQR and the results are compared to distinguish the better method and to evaluate the pros and cons of each method.

VI. LQR TUNING

The weighting matrices Q and R are tuned to obtain various responses and to study the behaviour of the system. The weights corresponding to the states and input variables are varied independently and the system is simulated in MATLAB to get the relevant responses. Generally the Q matrix is taken as $c^T c$ and the R matrix is taken as identity matrix. Fig. 4 shows the step responses of the system for various Q and R matrices.

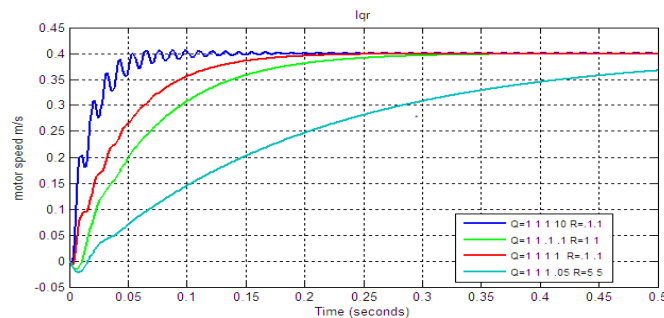
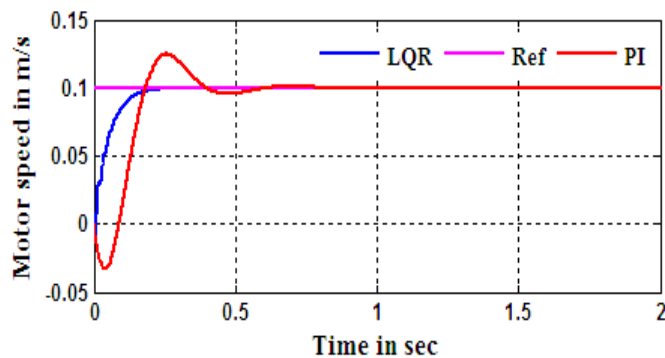


Fig 4 simulation result for proposed discrete LQR method by considering various Q and R matrices

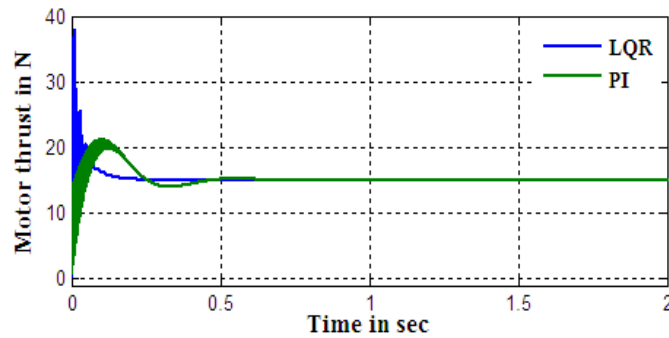
VII. SIMULATION RESULTS

Comparison of PI And Discrete LQR Methods

(a)The motor is operated by a speed command of step change of 0.1 m/s at thrust of 15N. comparison of speed & thrust are shown in Fig5



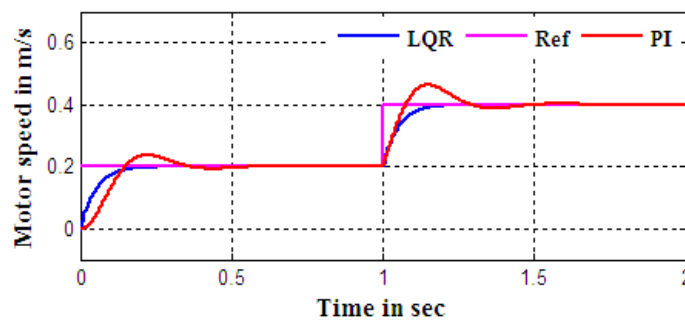
(a)



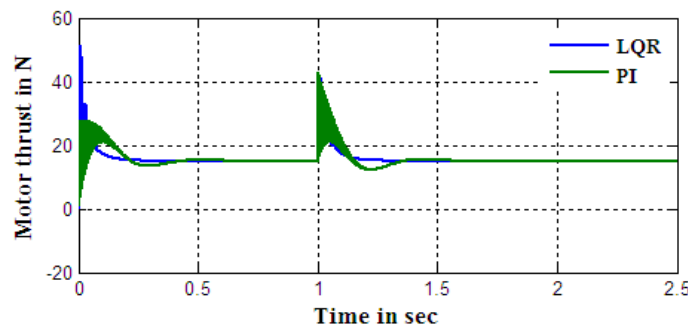
(b)

Fig.5 simulation result for the comparison of PI control method and discrete LQR method for step change in speed of 0.1 m/s at load thrust of 15N (a) motor speed in m/s (b) motor thrust in Newton

(b) The motor is operated by a speed command of step change of 0.2 m/s to 0.4m/s at thrust of 15N. The comparison of speed and thrust are shown in Fig. 6



(a)



(b)

Fig 6 simulation result for comparison of proposed discrete LQR method and conventional method for speed command of step change of 0.2 m/s to 0.4m/s at load thrust of 15N (a) motor speed in m/s (b) motor thrust in Newton

(c)The motor is operated by an trapezoidal speed command of 2m/s and an acceleration of 2 m/s² . the response of PMLSM is shown in Fig. 7

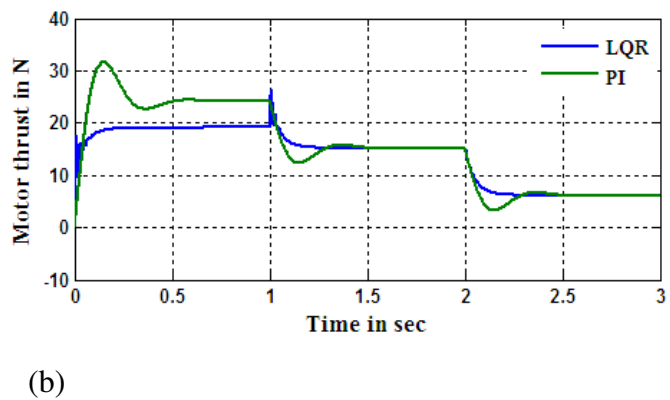
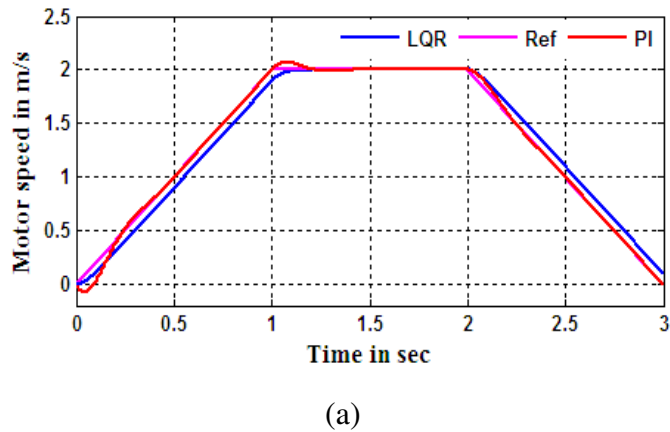
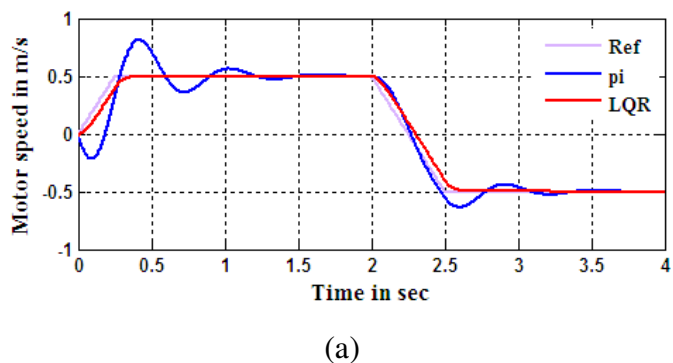
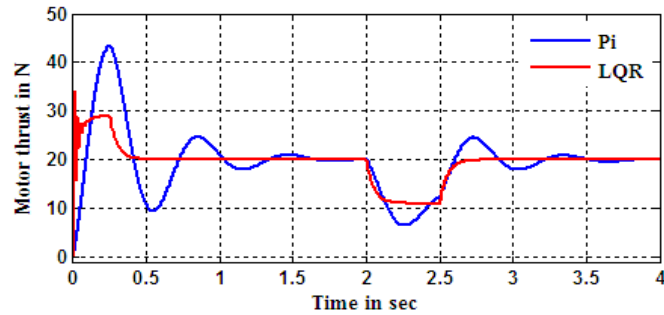


Fig. 7 simulation result for trapizoidal speed commad of 2m/s at an acceleration of 2m/s² at load thrust of 15N (a) motor speed in m/s (b)motor thrust in Newton

(d)The motor is operated in speed reversal operation of 2m/s and an acceleration of 2 m/s² . The response of PMLSM is shown in Fig. 8





(b)

Fig. 8 simulation results for speed reversal operation of 2m/s at an acceleration of 2m/s^2 at load thrust of 15N (a) motor speed in m/s (b) motor thrust in Newton

VIII. CONCLUSION

The dynamic equations of PMLSM are modelled and the non linear mathematical model of PMLSM is linearised using decoupling method. Discrete Linear quadratic Regulator is used to obtain the optimal state feedback gain matrix by minimising the objective function. The simulation results obtained from both conventional PI method and proposed method are compared in various conditions like step change in speed, trapezoidal input for high speed operations, step change in thrust and speed reversal operation and it is observed that the proposed discrete LQR control method has superior performance than the conventional PI control method.

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