Scientific Journal Impact Factor (SJIF): 3.518
International Journal of Modern Trends in Engineering and Research
IJMTER
ISSN (ONLINE) : 2349 - 9745
ISSN (PRINT) : 2393 - 8161

Reliability analysis of a redundant system with unreliable instrument and switch

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Abstract - In the paper we consider a redundant system consisting of n single-type elements one ofwhich is in the basic connection. There is also a switching device (SD) that realizes switching of nondefective redundant element in the basic connection if the element fails. Time of nonfailure operation and renewal of the elements and SD has exponential distribution. Distribution of nonfailure operation time of the basic element is an arbitrary distribution function. Probability of nonfailure operation and mean time of nonfailure operation of a redundant system is determined.

Keywords - redundant systems, switching device, probability of non failure operation, mean time of nonfailur operation.

I. INTRODUCTION

Natural and important way of reliability and longevity improvement of complex devices is to increase reliability of the elements composing these devices and this may be realized by different methods [9]. The most prospective solution of the reliability problem of technical systems is redundancy. The redundancy is that one or several redundant elements (or units) are joined to the element (or the unit) and at fault occurance they are successively plugged in place of the basic element (or a unit) and fulfil its functions. Availability of reserve components (or units), i.e. in other words, of the reserve essentially increases continuous operation interval of the system. In the simplest situations the solution on necessary reserve is not very complicated, but in work planning of a complicated system the solution of the matter what is to be reserved and in what extent grows out to an independent problem. When creating devices fulfilling important and responsible tasks it is necessary to attain maximum of reliability index involving constraints characterizing up-to-date feasibilities. These restraints are imposed for example, on price, and all restraints may be created by technological peculiarities of the process whose reliability should be provided [12]. Today there is a great member of papers devoted to this problem [1-6, 10-12].

1. A Redundant system and switching device without renewal (loaded reserve).

II. PROBLEM STATEMENT

Let us consider the following redundant system. There is n single-type elements, one of them is in the basic connection, $n-1$ elements are in the loaded reserve. There is also a switching device (SD) that realizes switching of nonfailure redundant element to the basic connection it the element in the base connection fails. The elements and SD function independently. Renewal of failed elements and SD is not performed. In availability of fault-free SD, switching is realized instantly.

Nonfailure operation time of the element has exponential distribution with parameter λ, nonfailure operation time of SD has exponential distribution with parameter λ_n.

Failure of the redundant system occurs either when all the elements included into the system fail, or as a result of failure of SD and then failure of the element in the base connection in the moment of SD fail.

It is required to find nonfailure operation probability of the reduandant system with SD and meantime of its nonfailure operation.

Now let us define these characteristics Begin with nonfailure operation probability $P(t)$. For $P(t)$ it holds the following formula [7]:

$$P(t)=e^{-\lambda_n t}\left(1-\left(1-e^{-\lambda t}\right)^n\right)+\int_0^t \lambda_n e^{-\lambda_n x}\left(1-\left(1-e^{-\lambda x}\right)^n\right)e^{-\lambda(t-x)}dx. \qquad (1)$$

Indeed, in order at moment t the system be operational, it is necessary:
A) either nonfailure operation of SD to the moment t and availability of at least one fault-free element at the moment t,
B) or availability of failure of SD on the interval $[0,t]$, nonfailure operation from the moment of failure of SD to the moment t of the element in the base connection at the moment of SD failure, and availability of a fault-free element (at least one) at the moment of SD failure.

These events A and B are inconsistent, therefore $\mathrm{P(t)=P(A)+P(Б)}$. The first summand in (1) is the probability of the event A, as the probability of fault-free operation of SD at the moment t is $e^{-\lambda_n t}$, probability of at least one fault-free element at the moment t is $\left(1-\left(1-e^{-\lambda t}\right)^n\right)$, and these probabilities are multiplied by virtue of independence of these events. The second summand in (1) is the probability of the event B. Indeed, the probability that SD will fail on the interval $(x, x+dx)$, $x+dx<t$, is $\lambda_n e^{-\lambda_n x}dx$, probability of availability of at least one fault-free element at the failure moment of SD is $1-\left(1-e^{-\lambda x}\right)^n$, and the probability of fault-free operation of the element in the base connection at failure moment of SD on the interval (x,t) is $e^{-\lambda(t-x)}$. As these three events are independent, then the probability of the event B is the integral from 0 to t from the product of probabilities of these three events. So, validity of formula (1) is established.

Transform P(B) to a more convenient form. We have

$$P(Á)=\lambda_n e^{-\lambda t}\int_0^t e^{(\lambda-\lambda_n)x}\left[1-\left(1-e^{-\lambda x}\right)^n\right]dx=\lambda_n\sum_{i=1}^n(-1)^{i-1}\frac{C_n^i}{\lambda_n+(i-1)\lambda}\left(e^{-\lambda t}-e^{-(\lambda_n+i\lambda)t}\right).$$

Thus, for $P(t)$ the following formula is valid:

$$P(t)=e^{-\lambda_n t}\left[1-\left(1-e^{-\lambda t}\right)^n\right]+\lambda_n\sum_{i=1}^n\frac{(-1)^{i-1}C_n^i}{\lambda_n+(i-1)\lambda}\left(e^{-\lambda t}-e^{-(\lambda_n+i\lambda)t}\right). \qquad (2)$$

For $\lambda_n=0$ i.e. when SD are absolutely reliable, we get the known formula
$P(t)=1-(1-e^{-\lambda t})^n$.

Now determine the mean time of nonfailure operation T_0 [4]. We have

$$T_0=\int_0^\infty P(t)dt=\int_0^\infty\left[e^{-\lambda_n t}\sum_{i=1}^n(-1)^{i-1}C_n^i e^{-\lambda i t}+\lambda_n\sum_{i=1}^n(-1)^{i-1}\frac{C_n^i}{\lambda_n+(i-1)\lambda}\left(e^{-\lambda t}-e^{-(\lambda_n+i\lambda)t}\right)\right]dt=$$

$$=\left(1+\frac{\lambda_n}{\lambda}\right)\sum_{i=1}^n(-1)^{i-1}\frac{C_n^i}{\lambda_n+i\lambda}. \qquad (3)$$

i.e. $T_0=\left(1+\frac{\lambda_n}{\lambda}\right)\sum_{i=1}^n(-1)^{i-1}\frac{C_n^i}{\lambda_n+i\lambda}$.

So, the following theorem is valid:

Theorem 1. For a redundant system consisting of n single-type element with SD without renewal at one element in the base connection and exponential distributions of nonfailure operation time and mean time of nonfailure operation of the elements and SD, its probability of non-failure operation and meantime of nonfailure operation are determined by the following formulas:

$$P(t)=e^{-\lambda_n t}\left[1-\left(1-e^{-\lambda t}\right)^n\right]+\lambda_n\sum_{i=1}^n(-1)^{i-1}\frac{C_n^i}{\lambda_n+(i-1)\lambda}\left(e^{-\lambda t}-e^{(\lambda_n+i\lambda)t}\right);$$

$$T_0 = \left(1+\frac{\lambda_n}{\lambda}\right)\sum_{i=1}^{n}(-1)^{i-1}\frac{C_n^i}{\lambda_n + i\lambda} ." \quad (4)$$

For $\lambda_n = 0$ we get the known result

$$T_0(\lambda_n = 0) = \frac{1}{\lambda}\sum_{i=1}^{n}(-1)^{i-1}\frac{C_n^i}{i} = \frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i} \quad (5)$$

For large n $T_0(\lambda_n = 0) \approx \frac{1}{\lambda}(\ln n + 0{,}577)$.

For a dublicated system without renewal $(n = 2)$.

$$P(t) = e^{-\lambda_n t}\left(2e^{-\lambda t} - e^{-2\lambda t}\right) + \lambda_n\left[\frac{2}{\lambda_n}\left(e^{-\lambda t} - e^{-(\lambda+\lambda_n)t}\right) - \frac{1}{\lambda+\lambda_n}\left(e^{-\lambda t} - e^{-(2\lambda+\lambda_n)t}\right)\right]. \quad (6)$$

$$T_0 = \left(1+\frac{\lambda_n}{\lambda}\right)\left(\frac{2}{\lambda+\lambda_n} - \frac{1}{2\lambda+\lambda_n}\right) = \frac{3\lambda+\lambda_n}{\lambda(2\lambda+\lambda_n)}.$$

In the case when $\frac{\lambda_n}{\lambda}$ may be considered as a small parameter, i.e. when $\lambda_n << \lambda$, this case is very often encountered in practice, a formula for calculation of T_0 may be simplified. Denoting $\frac{\lambda_n}{\lambda}$ by ε we have

$$\frac{1}{\lambda_n + i\lambda} = \frac{1/i\lambda}{1+\lambda_n/\lambda} = \frac{1}{i\lambda}\left(1 - \frac{\varepsilon}{i} + \frac{\varepsilon^2}{i^2} - \frac{\varepsilon^3}{i^3} + ...\right) = \frac{1}{i\lambda}\sum_{j=0}^{\infty}(-1)^j\frac{\varepsilon^j}{i^j},$$

$$T_0 = (1+\varepsilon)\sum_{i=1}^{n}(-1)^{i-1}\frac{C_n^i}{i\lambda}\sum_{j=0}^{\infty}(-1)^j\frac{\varepsilon^j}{i^j} = \frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i} + \frac{1}{\lambda}\sum_{j=1}^{\infty}\varepsilon^j(-1)^{j-1}\sum_{i=2}^{n}(-1)^{i-1}\frac{C_n^i(i-1)}{i^{j+1}}. \quad (7)$$

Hence, the first correction to the principal term of the expansion has the form $-\frac{\varepsilon}{2}\sum_{i=2}^{n}(-1)^i\frac{C_n^i(i-1)}{i^2}$, the second correction equals $-\frac{\varepsilon}{\lambda}\sum_{i=2}^{n}(-1)^i\frac{C_n^i(i-1)}{i^3}$, etc. i.e.

$$T_0 = \frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i} - \frac{\lambda_n}{\lambda^2}\sum_{i=2}^{n}(-1)^i\frac{C_n^i(i-1)}{i^2} + \left(\frac{\lambda_n}{\lambda}\right)^2\frac{1}{\lambda}\sum_{i=2}^{n}(-1)^i\frac{C_n^i(i-1)}{i^3} + \dots. \quad (8)$$

At practical calculations, we can use formula (8) already for $\varepsilon = \frac{\lambda_n}{\lambda} < 0{,}5$.

Now let us consider a redundant system with SD, that is the generalization of the above indicated system. The generalization is: In the base connection there is not one element but m elements, and these $n-m$ elements are in the loaded reserve. The same suppositions hold true for SD [3,4].

At first we consider the case when SD is absolutely reliable, i.e. $\lambda_n = 0$. Then probability of nonfailure operation of the system on the interval $(0,t)$ $P(t)$ is calculated by the formula

$$P(t) = \sum_{i=m}^{n} C_n^i e^{-\lambda i t}\left(1 - e^{-\lambda t}\right)^{n-i}, \quad (9)$$

and the meantime of nonfailure operation has the form

$$T_0 = \int_0^{\infty} P(t)dt = \sum_{i=m}^{n} C_n^i \int_0^{\infty} e^{-\lambda i t}\left(1-e^{-\lambda t}\right)^{n-i} dt = \frac{1}{\lambda}\sum_{i=m}^{n}\sum_{j=0}^{n-1}(-1)^j\frac{C_n^{i,j,n-i-j}}{i+j}, \quad (10)$$

where $C_n^{i,j,n-i-j} = \frac{n!}{i!\,j!(n-i-j)!}$.

For T_0 we can get a formula in another form. We have

$$T_0=\int_0^\infty\left[1-\sum_{i=0}^{m-1}C_n^i e^{-i\lambda t}\left(1-e^{-\lambda t}\right)^{n-i}\right]dt=\int_0^\infty\left(1-\sum_{i=0}^{m-1}C_n^i e^{-\lambda it}\sum_{j=0}^{n-i}C_{n-i}^j(-1)^j e^{-\lambda jt}\right)dt=$$

$$=\frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i}-\frac{1}{\lambda}\sum_{i=1}^{m-1}C_n^i\sum_{j=0}^{n-1}(-1)^j C_{n-i}^j\frac{1}{i+j}. \qquad (11)$$

The second term of this formula shows how much T_0 decreases when increasing the number of the elements in the base connection and respectively, decrease of the number of reserve elements [5,6]. Transform the formula for T_0 to a more convenient form. The following lemma is valid.

Lemma 1. It holds the equality

$$\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{1}{i+j}=\frac{1}{iC_n^i}". \qquad (12)$$

Proof. For $i=n$ the validity of equality (12) is checked directly. Now let $i<n$. Introduce the denotation

$$\varphi(n,i)=\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{1}{i+j}-\frac{1}{iC_n^i}.$$

Consider the difference $\Delta(n,i)=\varphi(n+1,i)-\varphi(n,i)$. It equals

$$\Delta(n,i)=\frac{(-1)^{n+1-i}}{n+1}-\frac{1}{iC_{n+1}^i}+\frac{1}{iC_n^i}+\sum_{j=0}^{n-i}\frac{(-1)^j}{i+j}\left(C_{n-i+1}^j-C_{n-i}^j\right)=$$

$$=\frac{(i-1)!(n-1)!}{(n+1)!}(n+1-n-1+i-i)-\varphi(n+1,i+1)=-\varphi(n+1,i+1).$$

Thus, $\varphi(n+1,i)-\varphi(n,i)=-\varphi(n+1,i+1)$ or $\varphi(n+1,i)=\varphi(n,i)-\varphi(n+1,i+1)$ (13)

In what follows, we use mathematical induction. The fact that $\varphi(n,n)=0$, for all n, was established above, hence induction may be used.

Substituting the relation $\varphi(n,n)=0$ into (13), we get $\varphi(n,n-1)=0$ for all n. Substituting this relation in (13), we get $\varphi(n,n-2)=0$ for all n. Constinueing in the same way, we get $\varphi(n,i)=0$ for all, and $i\le n$ and $0\le i\le n$, i.e. lemma 1 is proved.

Applying lemma 1 to formula (10) or (11), we find

$$T_0=\frac{1}{\lambda}\sum_{i=m}^{n}\frac{1}{i}. \qquad (14)$$

Now, let $\lambda_n>0$. For $P(t)$ the following formula is valid

$$P(t)=e^{-\lambda_n t}\sum_{i=m}^{n}C_n^i e^{-i\lambda t}\left(1-e^{-\lambda t}\right)^{n-i}+\int_0^t\lambda_n e^{-\lambda_n x}\sum_{i=m}^{n}C_n^i e^{-i\lambda x}\left(1-e^{-\lambda x}\right)^{n-i}e^{-m\lambda(t-x)}dx. \qquad (15)$$

The proof of validity of this formula is similar to the proof of validity of formula (1).

We can transform formula (15) to the following form

$$P(t)=e^{-\lambda_n t}\sum_{i=m}^{n}C_n^i e^{-i\lambda t}\left(1-e^{-\lambda t}\right)^{n-i}+\lambda_n\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{e^{-m\lambda t}-e^{-t[\lambda_n+\lambda(i+j)]}}{\lambda_n+\lambda(i+j-m)}. \qquad (16)$$

For the mean time of nonfailure operation T_0, in this case we have

$$T_0=\int_0^\infty P(t)dt=\left(1+\frac{\lambda_n}{m\lambda}\right)\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{1}{\lambda_n+\lambda(i+j)}. \qquad (17)$$

For $\lambda_n=0$ from (17) we get (10). Consequently, it holds the following theorem.

Теорема 2. For a reduandant system consisting of n single type elements, without SD without renewal at m element in the base connection and exponential distributions of nonfailure operation times and SD, its probability of nonfailure operation and meantime of nonfailure operation are determined by the formulas

$$P(t)=e^{-\lambda_n t}\sum_{i=m}^{n}C_n^i e^{-i\lambda t}\left(1-e^{-\lambda t}\right)^{n-i}+\lambda_n\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{e^{-m\lambda t}-e^{-t[\lambda_n+\lambda(i+j)]}}{\lambda_n+\lambda(i+j-m)},$$

$$T_0=\left(1+\frac{\lambda_n}{m\lambda}\right)\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{1}{\lambda_n+\lambda(i+j)}".$$

Theorem 2 is the generalization of theorem 1 for the case when the base connection contains m elements.

In the case when λ_n/λ may be considered as a small parameter, the formula for T_0 may be simplified. Denoting λ_n/λ by ε, we have

$$\frac{1}{\lambda_n+\lambda(i+j)}=\frac{1}{(i+j)\lambda}\sum_{l=0}^{\infty}(-1)^l\frac{\varepsilon^l}{(i+j)^l},$$

$$T_0=\left(1+\frac{\varepsilon}{m}\right)\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^j\frac{1}{(i+j)\lambda}\sum_{l=0}^{\infty}(-1)^l\frac{\varepsilon^l}{(i+j)^l}=$$

$$=\frac{1}{\lambda}\sum_{i=m}^{n}\frac{1}{i}+\frac{1}{m\lambda}\sum_{l=1}^{\infty}(-1)^{l-1}\varepsilon^l\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j(i+j-m)}{(i+j)^{l+1}} \tag{18}$$

Hence, the first correction to the principal term of the expansion has the form

$\frac{\varepsilon}{m\lambda}\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j(i+j-m)}{(i+j)^2}$, the second correction equals

$\frac{\varepsilon^2}{m\lambda}\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j(i+j-m)}{(i+j)^3}$ etc.

Then

$$T_0=\frac{1}{\lambda}\sum_{i=m}^{n}\frac{1}{i}-\frac{\varepsilon}{m\lambda}\left(-\sum_{i=m}^{n}\frac{1}{i}+m\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j}{(i+j)^2}\right)-$$

$$-\frac{\varepsilon^2}{m\lambda}\sum_{i=m}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j(i+j-m)}{(i+j)^3}+\ldots\ldots \tag{19}$$

From (4) and (17), (7) and (18) we get the following identities, assuming $m=1$,

$$\sum_{i=1}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j C_{n-i}^i\frac{1}{a+i+j}=\sum_{i=1}^{n}(-1)^{i-1}\frac{C_n^j}{a+i},\quad a>0 \tag{20}$$

$$\sum_{i=1}^{n}C_n^i\sum_{j=0}^{n-i}(-1)^j\frac{C_{n-i}^j(i+j-1)}{(i+j)^l}=\sum_{i=2}^{n}(-1)^{i-1}\frac{C_n^i(i-1)}{i^l},\quad n\ge 1,\ l\ge 2. \tag{21}$$

2. A redundant system and switching device without renewal (unloaded reserve).

Within the problem statement described in the previous section, we consider the case of unloaded reserve [9].

Let the base connection contain m elements.

For $\lambda_n=0$ the nonfailure operation probability $P(t)$ of the system for time t equals

$$P_0(t)=\sum_{i=0}^{n-m}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t} \tag{22}$$

For $\lambda_n>0$ for nonfailure operation probability $P_0(t)$ of the system for time t we have the following formula:

$$P(t)=e^{-\lambda_n t}\sum_{i=0}^{n-m}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}+\int_0^t \lambda_n e^{-\lambda_n x}\sum_{i=0}^{n-m}\frac{(m\lambda x)^i}{i!}e^{-m\lambda x}e^{-m\lambda(t-x)}dx. \qquad (23)$$

The proof of validity of this formula is similar to the proof of validity of formula (1). Performing simple transformations, we have:

$$P(t)=e^{-\lambda_n t}\sum_{i=0}^{n-m}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}+e^{-m\lambda t}\sum_{i=0}^{n-m}m^i\left(\frac{\lambda}{\lambda_n}\right)^i-e^{-t(m\lambda+\lambda_n)}\sum_{i=0}^{n-m}\left(\frac{\lambda}{\lambda_n}\right)^i m^i\sum_{k=0}^{i}\frac{(\lambda_n t)^{i-k}}{(i-k)!}. \qquad (24)$$

For mean time of nonfailure operation T_0 we find

$$T_0=\int_0^\infty P(t)dt=\int_0^\infty\left[\sum_{i=0}^{n-m}\frac{(m\lambda)^i}{i!}t^i e^{-t(m\lambda+\lambda_n)}+e^{-m\lambda t}\sum_{i=0}^{n-m}m^i\left(\frac{\lambda}{\lambda_n}\right)^i-\sum_{i=0}^{n-m}\left(\frac{\lambda}{\lambda_n}\right)^i m^i\sum_{k=0}^{i}\frac{\lambda_n^{i-k}}{(i-k)!}te^{i-k-t(m\lambda+\lambda_n)}\right]dt=$$

$$=\frac{1-\left(\frac{m\lambda}{m\lambda+\lambda_n}\right)}{\lambda_n}+\frac{1}{m\lambda}\left[1-\left(\frac{m\lambda}{m\lambda+\lambda_n}\right)^{n-m+1}\right]=\left(\frac{1}{\lambda_n}+\frac{1}{m\lambda}\right)\left[1-\left(\frac{m\lambda}{m\lambda+\lambda_n}\right)^{n-m+1}\right] \qquad (25)$$

From (22) T_0 for $\lambda_n=0$ equals $T_0=\frac{n-m+1}{m\lambda}$. Assuming in (25) $\lambda_n=0$, we get just this result.

Consider the case when the switch at the switching moment may in addition fail with probability α. This probability is stipulated by the fact that at the switching moment there may arise additional loadings on the switch , whereas to the switching moment it operates in standby mode of switching moment.

At first assume $\lambda_n=0$ and take into account $\alpha>0$. With probability $\alpha(1-\alpha)$ SD will fail on the k -th switch, under this condition nonfailure operation probability equals $\sum_{i=0}^{k-1}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}$ (as in this case the redundant elements in essence will be not $n-m$, but $k-1$), with probability $(1-\alpha)^{n-m}$ SD will not fail for all possible $n-m$ switches, and nonfailure operation probability, as was noted in formula (22) equals $\sum_{i=0}^{n-m}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}$. Then by the total probability formula, the nonfailure operation probability in this case equals

$$P_{\lambda_n=0}(t)=\sum_{k=1}^{n-m}\alpha(1-\alpha)^{k-1}\sum_{i=0}^{k-1}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}+(1-\alpha)^{n-m}\sum_{i=0}^{n-m}\frac{(m\lambda t)^i}{i!}e^{-m\lambda t}=\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)]^i}{i!}e^{-m\lambda t}. \qquad (26)$$

For $\lambda_n>0$ we have

$$P(t)=e^{-\lambda_n t}\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)]^i}{i!}e^{-m\lambda t}+\int_0^t\lambda_n e^{-\lambda_n x}\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)]^i}{i!}e^{-m\lambda x}e^{-m\lambda(t-x)}dx. \qquad (27)$$

After simple transformations we have

$$P(t)=e^{-\lambda_n t}\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)t]^i}{i!}e^{-m\lambda t}+e^{-m\lambda t}\sum_{i=0}^{n-m}\frac{[m(1-\alpha)\lambda]^i}{(\lambda_n)^i}e^{-t(m\lambda+\lambda_n)}\sum_{i=0}^{n-m}\left[\frac{\lambda m(1-\alpha)}{\lambda_n}\right]^i\sum_{k=0}^{i}\frac{(\lambda_n t)^k}{k!} \qquad (28)$$

In this case, for the mean time of nonfailure operation T_0 we have

$$T_0=\int_0^\infty P(t)dt=\int_0^\infty\left[\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)]^i}{i!}t^i e^{-t(m\lambda+\lambda_n)}+e^{-m\lambda t}\sum_{i=0}^{n-m}\left[\frac{m\lambda(1-\alpha)}{\lambda_n}\right]^i-e^{-t(m\lambda+\lambda_n)}\sum_{i=0}^{n-m}\left[\frac{m\lambda(1-\alpha)}{\lambda_n}\right]^i\times\right.$$

$$\left.\times\sum_{k=0}^{i}\frac{\lambda_n^k}{k!}t^k\right]dt=\frac{m\lambda+\lambda_n}{m\lambda(m\lambda\alpha+\lambda_n)}\left[1-\left(\frac{m\lambda(1-\alpha)}{m\lambda+\lambda_n}\right)^{n-m+1}\right] \qquad (29)$$

$$T_0 = \frac{m\lambda + \lambda_n}{m\lambda(m\lambda\alpha + \lambda_n)}\left[1 - \left(\frac{m\lambda(1-\alpha)}{m\lambda + \lambda_n}\right)^{n-m+1}\right].$$

So, the following theorem is valid:

Theorem 3. For a redundant system with m elements in the base connection and unloaded reserve in the simplest suppositions, allowing for possibility of failure of SD both to the switching moment and at the switching moment itself, $P(t)$ and T_0 have the form.

$$P(t) = e^{-(m\lambda+\lambda_n)t}\sum_{i=0}^{n-m}\frac{[m\lambda(1-\alpha)t]^i}{i!} + \frac{\lambda_n e^{-m\lambda t}}{\lambda_n - m(1-\alpha)t}\left[1 - \left(\frac{m\lambda(1-\alpha)}{\lambda_n}\right)^{n-m+1}\right]\frac{\lambda_n e^{-(m\lambda+\lambda_n)t}}{\lambda_n - m\lambda(1-\alpha)}\times$$

$$\times\sum_{k=0}^{n-m}\frac{[m\lambda(1-\alpha)t]^k}{k!}\left[1 - \left(\frac{m\lambda(1-\alpha)}{\lambda_n}\right)^{n-m-k+1}\right];$$

$$T_0 = \frac{m\lambda + \lambda_n}{m\lambda(m\lambda\alpha + \lambda_n)}\left[1 - \left(\frac{m\lambda(1-\alpha)}{m\lambda + \lambda_n}\right)^{n-m+1}\right].$$

The obtained results generalize the results reduced in [8].

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