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ONTERNARY QUADRATIC DIOPHANTINE EQUATION $2x^2 + 3y^2 = 4z$

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Abstract-The ternary quadratic diophantine equation $2x^2 + 3y^2 = 4z$ is analyzed for its non-zero distinct integral points on it. A few interesting properties among the solutions are presented.

Keywords: Integral points, Ternary quadratic, Polygonal numbers, Pyramidal numbers and Special numbers.

Mathematics subject classification number: 11D09

Notation:

$t_{m,n}$ = Polygonal number of rank n with sides m

$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

g_n = Gnomonic number

wo_n = Woodhal number

mer_n = Mersenne number

I. INTRODUCTION

Diophantine equations is an interesting concept, as it can be seen from [1-2]. For an extensive review of various problems one may refer[3-11]. In this context one may also see [12-23]. This communication concerns with yet another interesting ternary quadratic diophantine equation $2x^2 + 3y^2 = 4z$ for determining its infinitely many non-zero integral solutions. Also a few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The ternary quadratic equation to be solved for its non-zero integral solution is

$$2x^2 + 3y^2 = 4z \quad (1)$$

Assume

$$x = 2X , y =2Y \quad (2)$$

On substituting (2) in (1) , we get

$$2X^2 + 3Y^2 = z \quad (3)$$

Let $X = \alpha+3\beta , Y = \alpha-2\beta , z = 5\gamma^2$ (4)

Substituting (4) in (3) , we get

$$\alpha^2 + 6\beta^2 = \gamma^2 \quad (5)$$

Where α , β and Υ are non-zero integers.

Different patterns of solution for (1) are given below

PATTERN:1

Equation (5) can be written as

$$\alpha^2 + 6\beta^2 = \gamma^2 * 1 \tag{6}$$

Write 1 as

$$1 = \frac{(1+i2\sqrt{6})(1-i2\sqrt{6})}{25} \tag{7}$$

Assume

$$\gamma = a^2 + 6b^2 \tag{8}$$

Where a, b are non-zero distinct integers.

Substituting (7) & (8) in(6), we get

$$\alpha^2 + 6\beta^2 = (a^2 + 6b^2)^2\left[\frac{(1+i2\sqrt{6})(1-i2\sqrt{6})}{25}\right]$$

On employing the method of factorization and on equating real and imaginary parts, we get

$$(\alpha + i\sqrt{6}\beta)(\alpha - i\sqrt{6}\beta) = (a + i\sqrt{6}b)^2 \ (a - i\sqrt{6}b)^2\left[\frac{(1+i2\sqrt{6})(1-i2\sqrt{6})}{25}\right]$$

On comparing the positive and negative factors, we get

$$(\alpha + i\sqrt{6}\beta) = (a + i\sqrt{6}b)^2\left[\frac{1+i2\sqrt{6}}{5}\right] \tag{9}$$

$$(\alpha - i\sqrt{6}\beta) = (a - i\sqrt{6}b)^2\left[\frac{1-i2\sqrt{6}}{5}\right] \tag{10}$$

On comparing the rational and irrational parts from the above equation, we get

$$\alpha = \frac{1}{5}(a^2 - 6b^2 - 24ab) \tag{11}$$

$$\beta = \frac{1}{5}[2(a^2 - 6b^2 + ab)] \tag{12}$$

As our interest is to find only integer solution, it is seen that α, β are integers for suitable choices of a and b.

Let us assume a = 5A and b = 5B in (11) and (12), we get

$$\alpha = 5A^2 - 30B^2 - 120AB \tag{13}$$

$$\beta = 10A^2 - 60B^2 + 10AB \tag{14}$$

Substituting (13) and (14) in (2), thenon-zero distinct integral solutions of (1), we get

$$x = x(A, B) = 2(35A^2 - 210B^2 - 90AB)$$

$$y = y(A, B) = 2(-15A^2 + 90B^2 - 140AB)$$

$$z = z(A, B) = 5(25A^2 + 150B^2)^2$$

PROPERTIES:

1. $x(A,1) + y(A,1) - t_{42,A} - 4t_{12,A} \equiv -240 \pmod{425}$
2. $z(A,1) = 250\left(ct_{25,A^2} + 55ct_{5,A}\right) - 5(3437g_A + A + 16263)$

PATTERN:2

Write (5) as,

$$\gamma^2 - \alpha^2 = 6\beta^2$$

$$(\gamma+\alpha)\ (\gamma-\alpha)\ =\ 6\beta^2 \quad (15)$$

Case 1:

$$\frac{\gamma+\alpha}{6\beta} = \frac{\beta}{\gamma-\alpha} = \frac{p}{q} \quad (16)$$

This is equivalent to the following two equations

$$q\alpha - 6p\beta + q\gamma = 0$$
$$p\alpha -+q\beta - p\gamma = 0$$

Applying the method of cross multiplication, we get

$$\left.\begin{array}{l} \alpha = \alpha(p, q) = 6p^2 - q^2 \\ \beta = \beta(p, q) = 2pq \\ \gamma = \gamma(p, q) = 6p^2 + q^2 \end{array}\right\} \quad (17)$$

Substituting the values of (17) in (2), the non-zero distinct integral values of x,y and z satisfying (1) are given by

$$x = x(p, q) = 2\ (6p^2 - q^2 + 6pq)$$
$$y = y(p, q) = 2\ (6p^2 - q^2 - 4pq)$$
$$z = z(p, q) = 5\ (36p^4 + q^4 + 12p^2q^2)$$

PROPERTIES:

1. $x(1,q) - y(1,q) + z(1,q) = ct_{10,q^2} + t_{112,q} + 37g_q + 216$
2. $x(n,2n+1) - y(n,n+1) - 41n = t_{54,n}$

Case 2:

Equation (15) can be rewritten as

$$\frac{\gamma+\alpha}{\beta} = \frac{6\beta}{\gamma-\alpha} = \frac{p}{q}$$

On following the procedure as in case (1) the non-zero distinct solutions of (1) are given by

$$x = x(p, q) = 2\ (p^2 - 6q^2 + 6pq)$$
$$y = y(p, q) = 2\ (p^2 - 6q^2 - 4pq)$$
$$z = z(p, q) = 5\ (p^4 + 36q^4 + 12p^2q^2)$$

PROPERTIES:

1. $-\frac{x(2^n,n)}{2} + Mer_{2n} + 6wo_n + 7 =$ Nasty number
2. $x(2^n,2^n) - y(2^n,2^n) = 10(car1_n + ky_n) + 20$

PATTERN:3

Rewrite (5) as,

$$\gamma^2 - 6\beta^2 = \alpha^2 * 1 \quad (18)$$

Write 1 as,

$$1 = (5 + 2\sqrt{6})(5 - 2\sqrt{6}) \quad (19)$$

Assume

$$\alpha = a^2 - 6b^2 \quad (20)$$

Using (19) and (20) in (18), we get

$$\gamma^2 - 6\beta^2 = \left(a^2 - 6b^2\right)^2 \left(5 + 2\sqrt{6}\right)\left(5 - 2\sqrt{6}\right)$$

On employing the method of factorization and equating the positive and negative factors, we get

$$\left.\begin{aligned} \alpha &= a^2 - 6b^2 \\ \beta &= 2a^2 + 12b^2 + 10ab \\ \gamma &= 5a^2 + 30b^2 + 24ab \end{aligned}\right\} \quad (21)$$

Substituting the values of (21) in (2), the non-zero distinct integral values of x,y and z satisfying (1) are given by

$$x = x(a, b) = 2\,(7a^2 + 30b^2 + 30ab)$$
$$y = y(a, b) = 2\,(-3a^2 - 30b^2 - 20ab)$$
$$z = z(a, b) = 5\,(5a^2 + 30b^2 + 24ab)^2$$

PROPERTIES:

1. $x(a+1,1) + y(a+1,1) - \left(t_{8,a} + t_{12,a} + 21g_a\right) = 49$
2. $y(n+3,-2n) = 2\left(-t_{68,n} - t_{62,n} - t_{42,n} + 11g_n - 16\right)$

III. CONCLUSION

One may search for other patterns of solution and their corresponding properties.

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