

Modeling of Stochastic Vertical Stationary Transport Systems

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Abstract - We consider mathematical models stochastic vertical transportation lift systems with arbitrary values of customer intensity. Such systems have essentially more complicated structure and their investigations by analytical approaches faces with some troubles. Our aim is to introduce the various control strategies for these systems, compare them and find the advantage of different policies. By modeling a behavior of such systems on a computer and getting numerical results for different efficiency indexes and characteristics we can compare the various control policies and to derive the optimal among them.

Keywords - vertical transportation, lift, characteristics, waiting time, customer, simulation

I. INTRODUCTION

In the second part of the XX century the methods and approaches of queuing theory have been applied in investigating of traffic flows, elevators and escalators systems, airports, control of traffic flows in tunnels, shipping, transportation, communication systems, network of computers and others [1,2,3,9]. All of these systems are unified in one common idea – queues with moving servers. Investigation of such types of queues leads to a construction and research of new mathematical models and in the frames of such mathematical models the new methods of investigations are formed. Simulation, modeling, intensive methods of computational statistics and other modern IT methods for queuing systems are a wide field of investigations, but even now there are a lot of unsolved problems, for instance with simulation of rare events, observation of traffic jams, different car accidents and other, which are happened with a small probability. Perhaps, even modeling will not allow the observation of rare events, because sometimes for their observing it is necessary to have long period of modeling and it is difficult to estimate how much time is needed for modeling of rare events.

One of the main tools for investigation of complicated queuing systems is empirical and simulation data and their analysis. By simulation we can get numerical results of the different characteristics of systems. But for taking decisions that are rich in content, it is necessary to have statistical analysis of simulation data [4,5,10,11].

Complex usage of analytical and computer methods allow taking a rich in content decisions for queues with complicated structures and making corresponding recommendations for practical applications.

Typical examples of queues with moving servers are elevator systems[11-13]. In 1953 there was an opening ceremony of the new building of M.V.Lomonosov Moscow State University, where there were three big elevator halls, with six elevators in each. In one of the elevator hall some elevators went up to 1,2,...n/2 floors (n is a number of the floors) and others went up to 1, (n/2)+1,(n/2)+2,..., n floors. (see, Picture 1, elevator hall in Moscow State University)

The great Kolmogorov, who participated in the opening ceremony immediately pointed out, that it is the wrong control policy and instead of the 1, (n/2)+1,(n/2)+2,..., n, floors he suggested to use

1,2,...,2n/3. But with the use of the modeling of the behavior of elevator systems on the computer, was shown that the Kolmogorov's advice was correct [12]. The alternative to such control can be a system where one elevator serves even numbered floors and the other one serves odd numbered floor. However, the modeling on the computer showed that in this system the average waiting time of a passenger is greater than the average waiting time of a passenger in the system where one elevator serves 1, 2n/3, (2n/3)+1, ... , n.

One other interesting unofficial control policy was used in the students' dormitory of Lomonosov Moscow State University, which was called "higher – lower". There were 18 floors in the student dormitory and four elevators operated between the 1st to 12th, 14th, 16th and 18th floors (for the elevators to work more rapidly they skipped the odd numbered floors, after 12th), there was another elevator hall for service between the 1st-10th floors. If an elevator came to the first floor and the first student yelled the word "HIGHER" then the elevator would be used for going up only to the higher floors (16th and 18th) and next elevator will give service to the 12th, 14th and other floors. If the first call would have been "LOWER", then the elevator would operate between the lower floors (12th, 14th and other floors). (see, picture 3, student dormitory in MSU).

The mathematical models and modeling this strategy on a computer, which confirmed an advantage of the "higher – lower" strategy were constructed and investigated [12,13].

The construction and investigation of mathematical models of queues with delays as a control function gave unexpected results. Introduction of delays, "HIGHER-LOWER" and others control policies in queues, allowed the diminishment of expectation of waiting time of customers before service in some models.

There is another control policy by elevators system "EVEN-ODD", when one elevator gives service only for "EVEN" and other for "ODD" floors. Comparing of "HIGHER-LOWER" with "EVEN-ODD" showed an advantage of "HIGHER-LOWER" control policy.

Construction of mathematical models of such systems leads needs using of some deep mathematical conception and facts from theory of point processes [14,15]. Some control problems by these systems were investigated in [6,7,8,16].

In the paper various control policies by elevator system are investigated. By simulation it was derived an advantage of one system in comparison with other. Numerical examples of simulation and graphs, describing of behavior of various parameters are given.

II.GENERAL MATHEMATICAL MODEL

Let us consider the system $L_k F_n C_{xx}$, i.e. k lifts, which serve customers in the building with n floors and with control policy xx (Fig. 1). The flow of customers, which arrives for service, is stationary. Remind that $\lambda_{i,j}$ ($\lambda_{ii} = 0; i, j = 1,2,...,N$) is the intensity of customers who goes from i -th floor to the j -th.

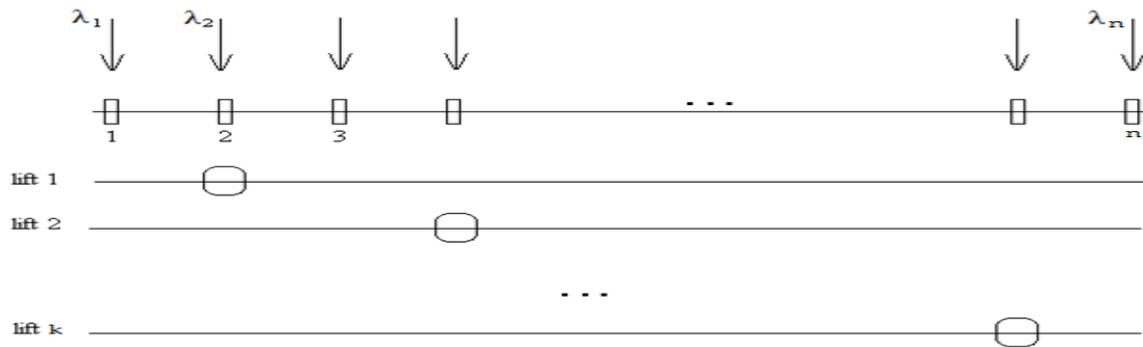


Fig.1

There are different types of systems. Introduce so called “collecting systems”.

1. Collecting service systems.

If there is customer at the i -th floor (going up) and lift goes from j -th floor ($j < i$, i.e. lift goes from down to up), then lift must check existence of customers (going up) between j -th and i -th floor and takes them. Similarly, if there is customer at the i -th floor (going down) and lift goes from j -th floor $j > i$ (from up to down) then lift must collect customers (going down) between j -th and i -th floor. Such systems with unbounded volume have some deficiency, because in saturated regime (high intensity of customer flow at the each floor) some customers will get service much more later, than customers who arrived into the system later?

2. Direct service systems (non-collecting systems).

If there is a customer at the i -th floor (going up) and lift goes from j -th floor ($j < i$, i.e. lift goes from down to up), then in spite of that there are customers (going up) between i -th and j -th floor lift directly goes to i -th floor, i.e. lift ignores customers who arrived into the system later than customer at the i -th floor. It is a like to FIFO system, because the principle first income first outcome is realized here. If there is a customer at the i -th floor (going down) and lift goes from j -th floor $j > i$ (from up to down) then lift ignores all customers between j -th and i -th floor until the instant while customers from i -th floor get service. Such lift systems can be observed in some not so high living buildings.

3. Mixed systems. If lift goes from up to down it collects customers at the below floor. But moving from down to up lift ignores customers while customer in lift gets service.

Second and third systems are widely used today in the not so high living buildings, but in skyscrapers are usually used “collecting systems” with sufficiently large volume, i.e. first system.

In modern living buildings there are exist various complicated control policies by such systems. For instance, if there are two lifts and both are going to down then collection of customers will be distributed between lifts. The lift system gives commands to lifts, which floors should be served and which floors should be ignored. Arrival of any new customer into the system leads to the correction of control policy, i.e. to the new commands. Such systems also will be considered below.

III. SIMULATION

For simulation of the systems the following notations are introduced.

- i -a current number of customer, which arrived and into the system and getting service;
- j -a number of the floor, where i -th customer arrived;
- k -a number of the floor, where customer is going out;
- t_i -an instant, when i -th customer arrived into the system;
- l -an order number of the lift, which gives service for i -th customer;
- t_i^{ss} -a starting service instant for i -th customer(an instant, when i -th customer will get a lift);
- t_i^{sf} -an instant, when i -th customer finishes service (leaves lift).

For i -th customer we will introduce 7-dimensional vector $\{i, j, k, t_i, l, t_i^{ss}, t_i^{sf}\}$, which has been saved in archive and allows to compute desired efficiency indexes. During a simulation the archive of the 7-dimensions vectors will be created and using the components of that 7-dimensional vector the efficiency indexes of the systems can be calculated. For instance, a customer average waiting time can be calculate as

$$\sum_{i=1}^N (t_i^{ss} - t_i) / N$$

where N is a number of all served customers, i.e. number of all the 7-dimensional vectors. An average service time can be calculated as

$$\sum_{i=1}^N (t_i^{sf} - t_i^{ss}) / N$$

and so on.

For simulation the following values of technical characteristics were used.

$h[k] = (k-1)h_1, h_1 = 3\text{sec.}$

$h_2 = 2\text{sec.}$ - stopping time interval at the floor (opening and closing a door)

$m = 5$ (the volume of lift, maximum number of people in the lift).

We will consider several cases:

The system $L_2 F_{21} C_{IL}$, where IL – means non-controlled policy, i.e. lifts operating independently.

The system $L_2 F_{21} C_{AA}$, where AA – means controlled policy, i.e. the nearest lift serves customer.

The system $L_2 F_{21} C_{DD}$, where DD – means distributed policy, i.e. floors are distributed between lifts. There are different ways of distribution:

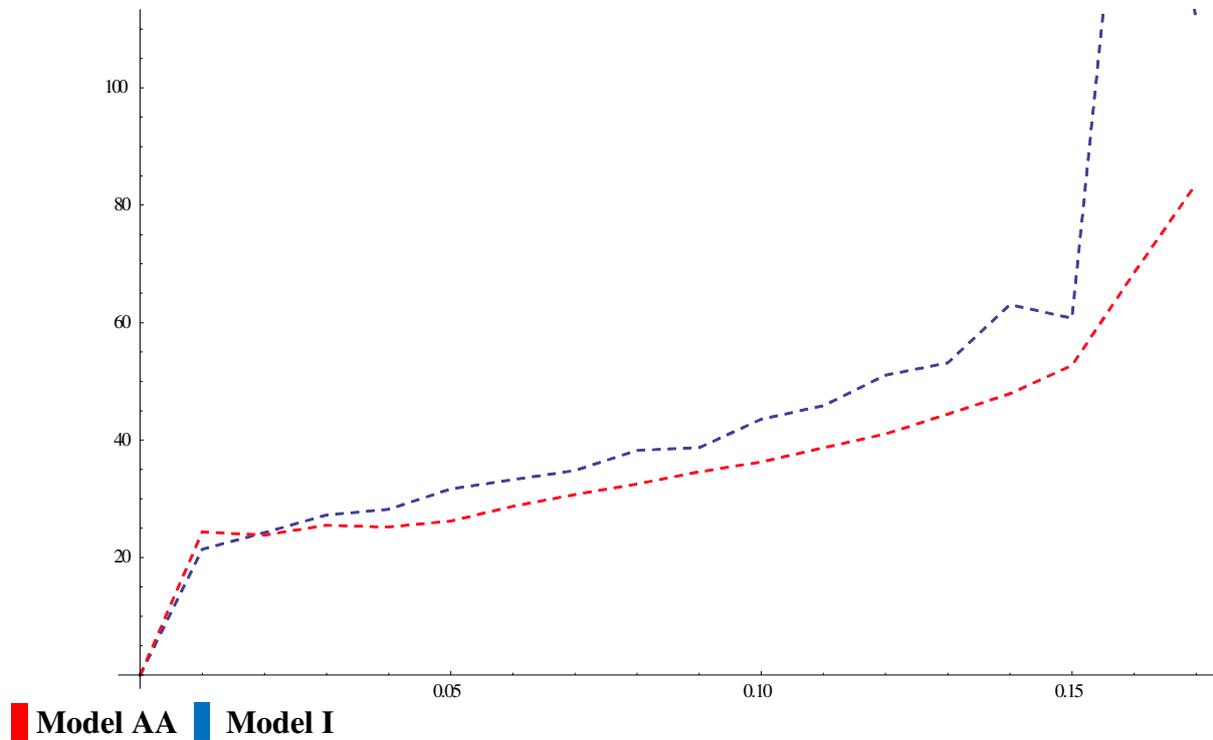
DD1 – first lift serves odd numbered floors, the second lift serves even numbered floors;

DD2 – serving interval of lifts are determined by customers in the first floor at random, if low floors are chosen lift serves floors 1 - $2N/3$ and if “high floors” are chosen lift serves floors $2N/3 - N$.

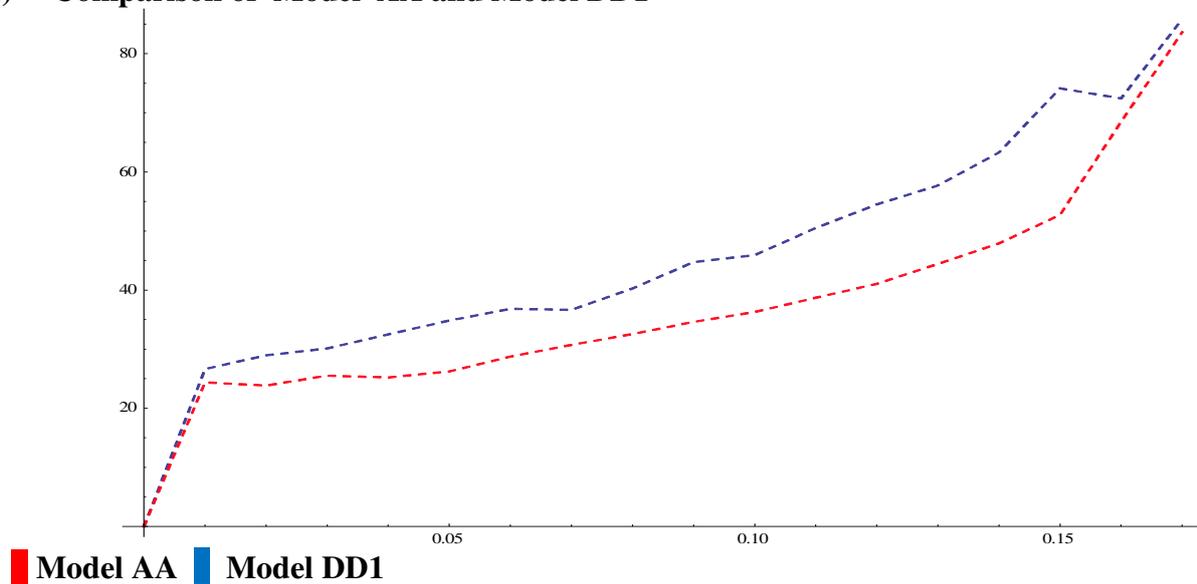
DD1 – first lift serves floors 1 - $2N/3$, the second lift serves floors $2N/3 - N$;

III. EXPERIMENTAL RESULTS

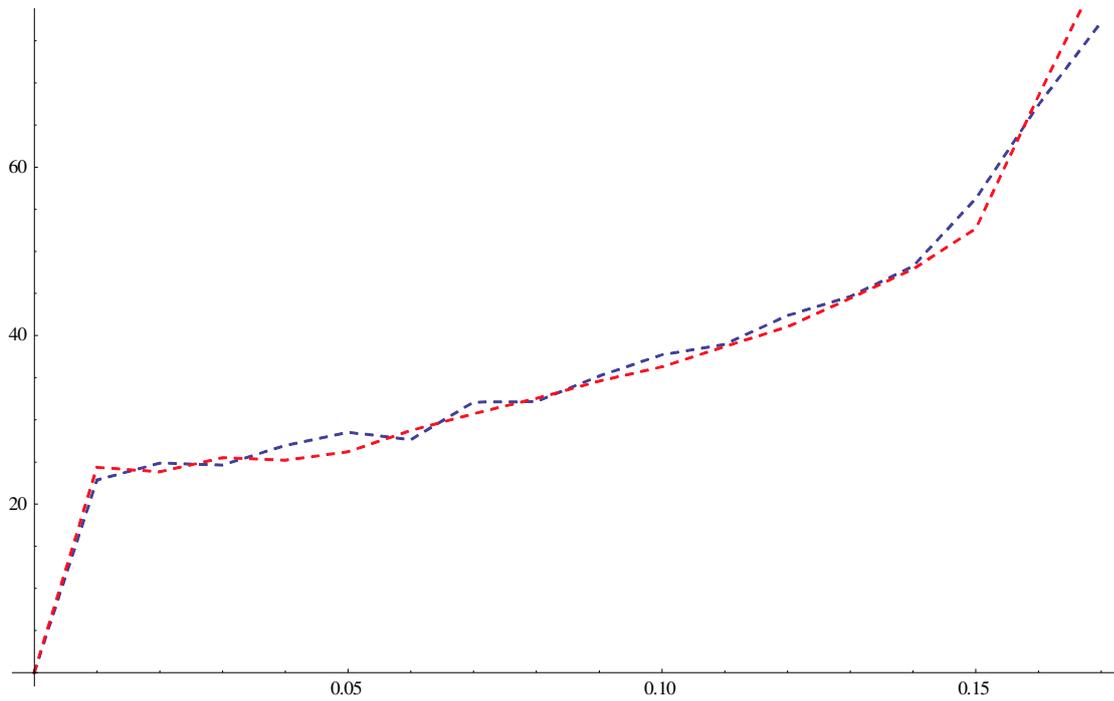
- 1) For low values of intensity ($0 \leq \lambda \leq 0.17$) average waiting time of customers - $W = W(\lambda)$
- 1.1) Comparison of Model AA and Model IL



- 1.2) Comparison of Model AA and Model DD1

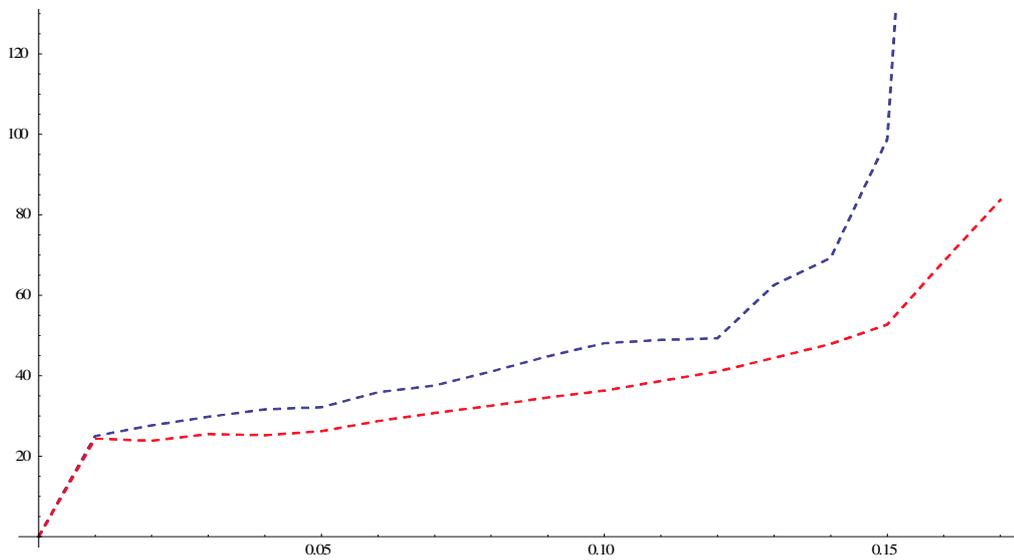


1.3) Comparison of Model AA and Model DD2



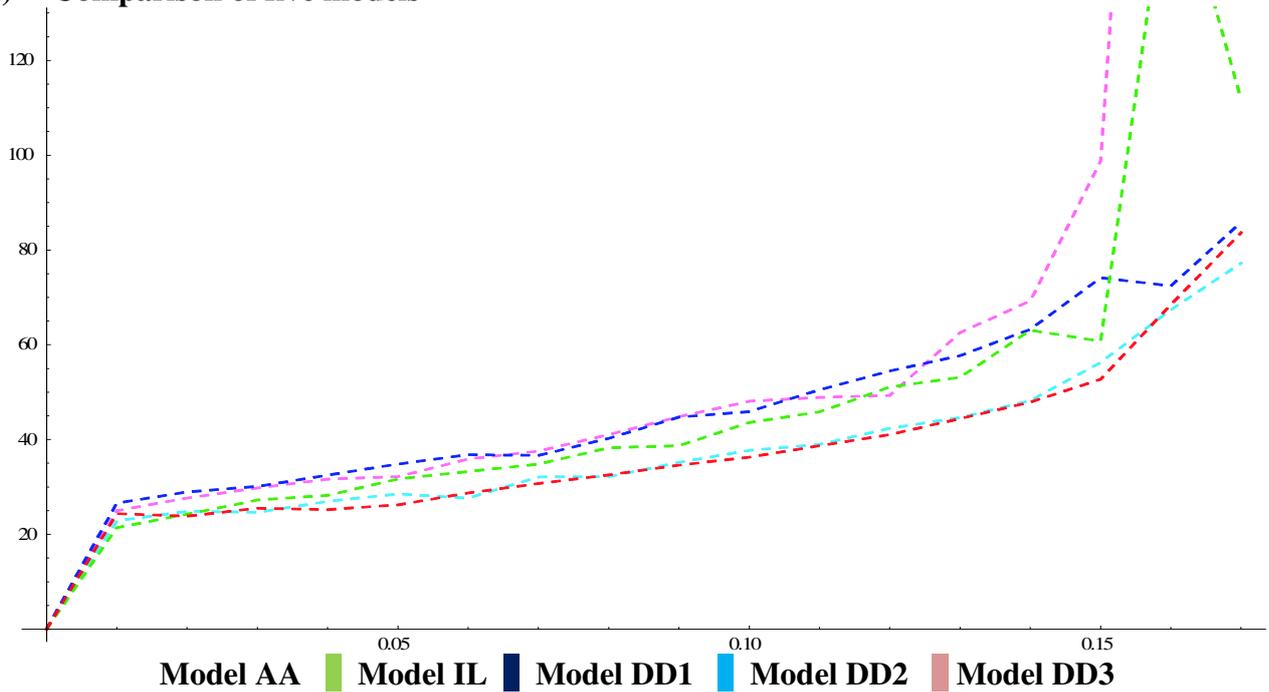
■ Model AA ■ Model DD2

1.4) Comparison of Model AA and Model DD3



■ Model AA ■ Model DD3

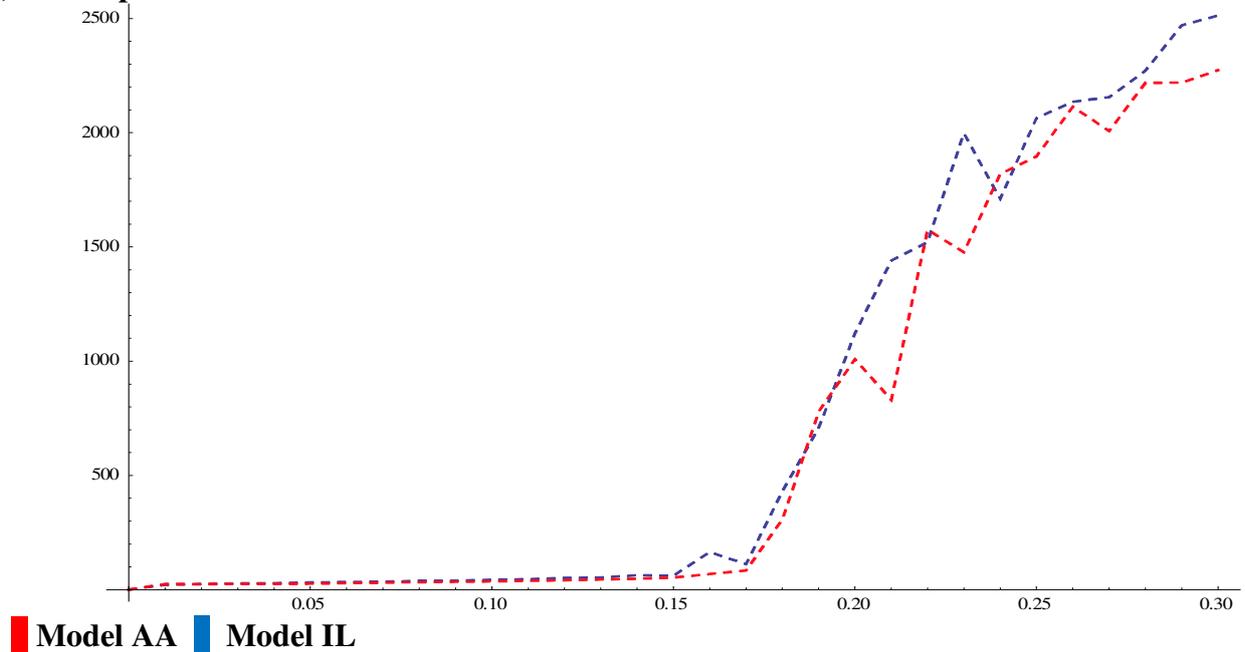
1.5) Comparison of five models



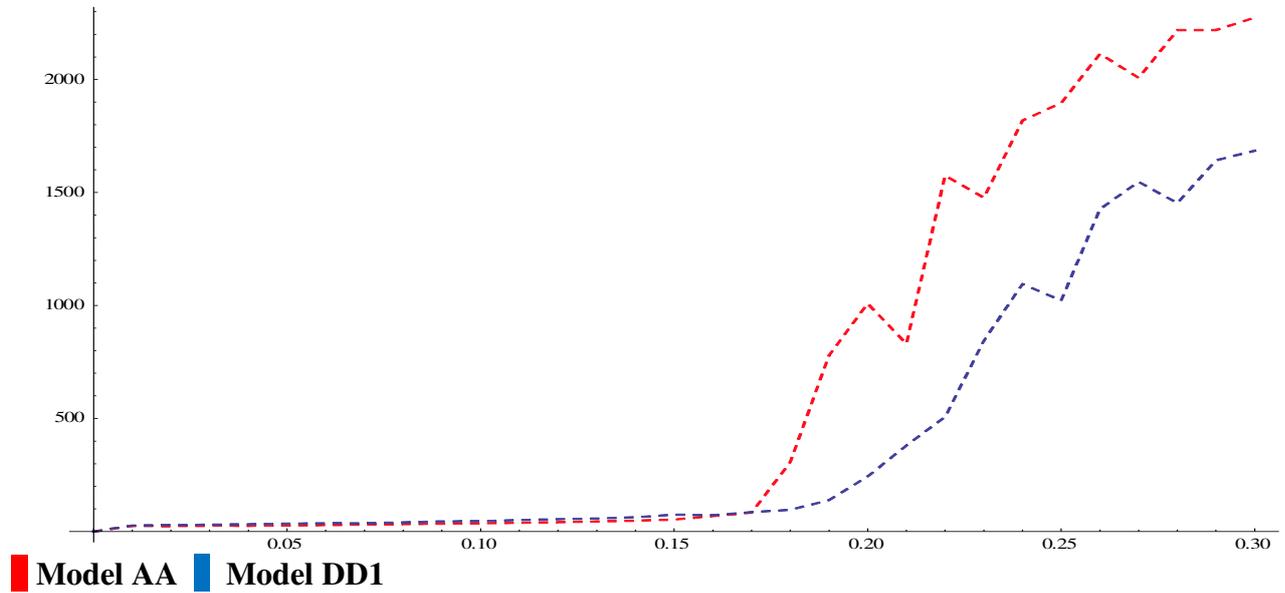
As we see from experimental results for low values of intensity Model AA performs better than others.

2) For high values of intensity ($0.18 \leq \lambda \leq 0.3$) average waiting time of customers - $W = W(\lambda)$

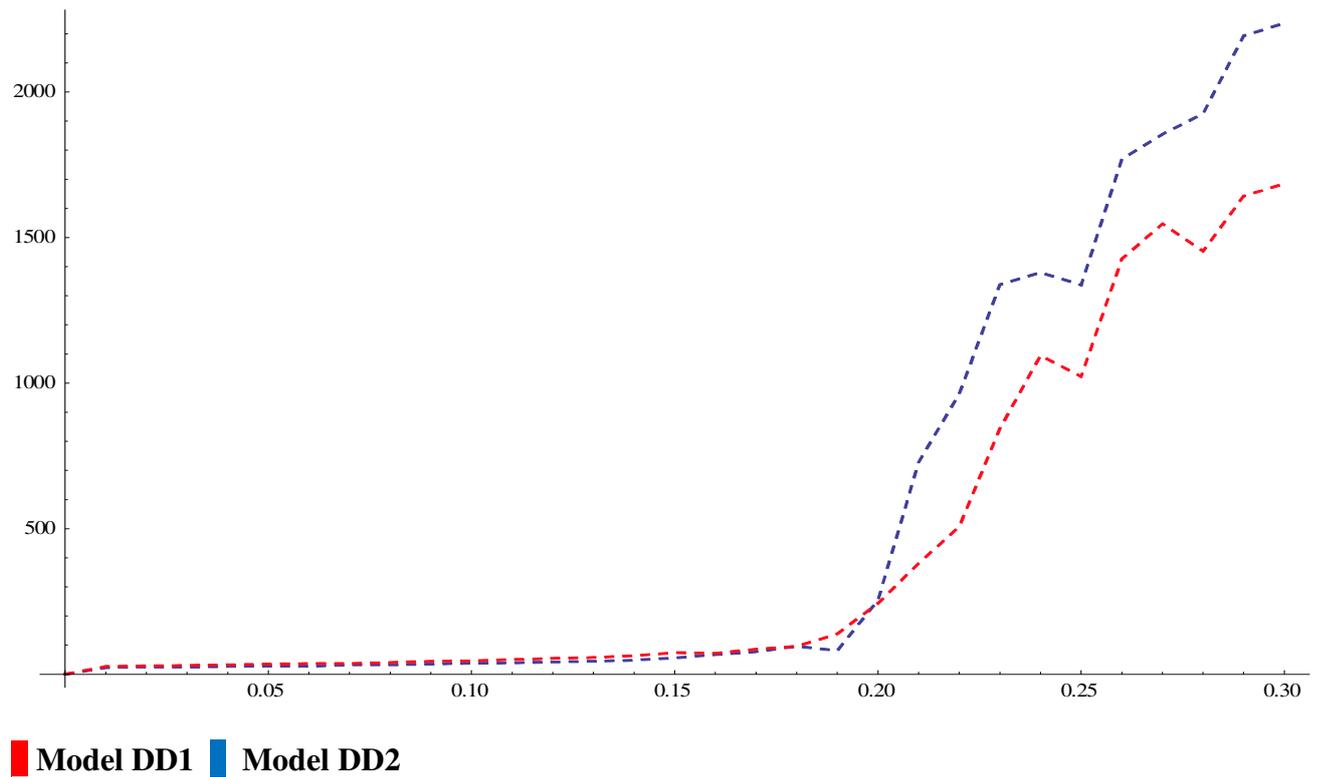
2.1) Comparison of Model AA and Model IL



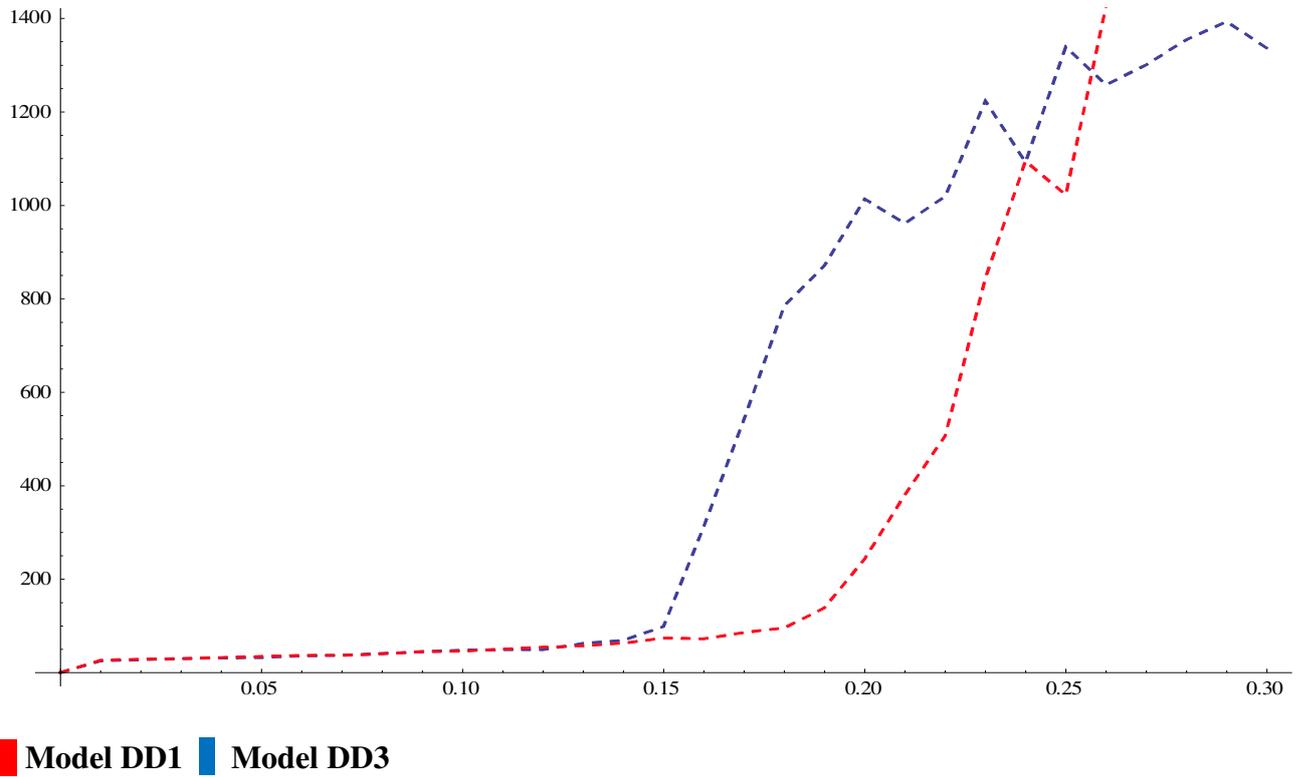
2.2) Comparison of Model AA and Model DD1



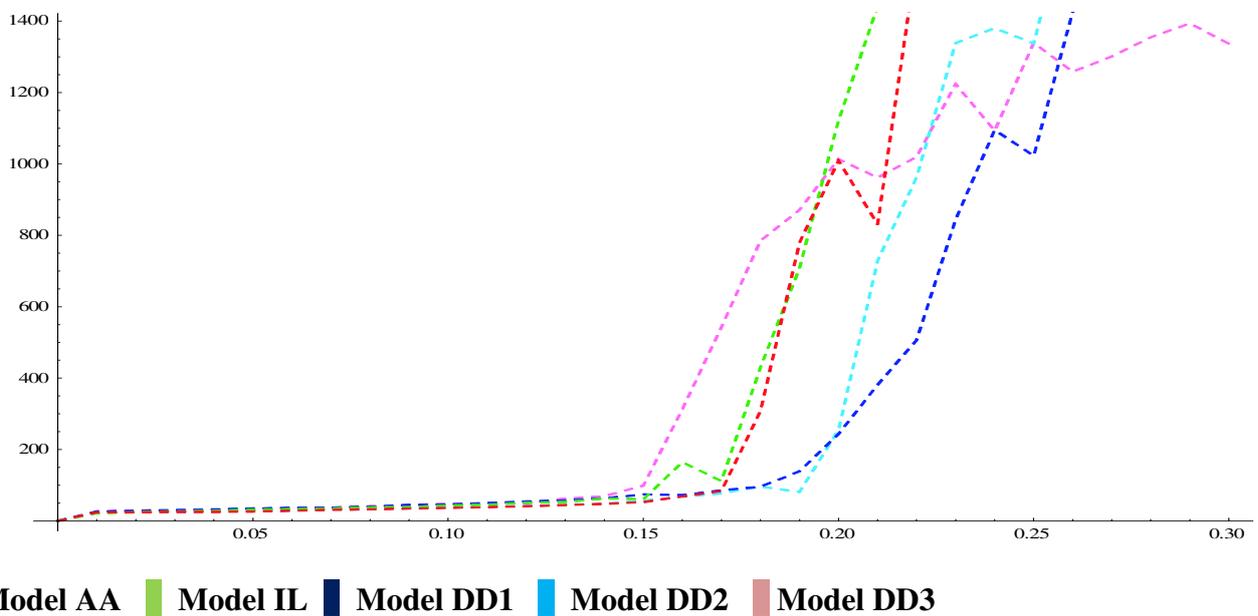
2.3) Comparison of Model DD1 and Model DD2



2.4) Comparison of Model DD1 and Model DD3



2.5) Comparison of five models



As we see from experimental results for low values of intensity Model DD1 performs better than others.

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