Scientific Journal Impact Factor (SJIF) : 3.518
International Journal of Modern Trends in Engineering and Research
ISSN (ONLINE) : 2349 - 9745
ISSN (PRINT) : 2393 - 8161

Modelling the Impact of Multi-Interventions Campaigns on Lymphatic Filariasis Disease

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Abstract—In this paper, a deterministic Lymphatic Filariasis (LF) model is formulated and analysed with the aim of assessing the impact of multi-intervention campaigns on transmission dynamics of Lymphatic Filariasis disease. The role of Mass Drug Administration (MDA), health education campaign (on the use of Insecticides Treated Nets (ITN), Indoor Residual Spraying (IRS) and environmental cleanliness) to community and Sterile Insect Technique (SIT) is established. Qualitative analysis is implemented to determine the basic reproduction number R_e necessary for the control of the disease in communities. It is observed that, the disease-free equilibrium (DFE) exists and is locally and globally asymptotically stable if $R_e < 1$ whereas if $R_e > 1$ the endemic equilibrium exists and it is globally asymptotically stable. Numerical simulations are performed to complement the analytical results.

Keywords—Mass Drug Administration (MDA), Health Education Campaigns, Sterile Insect Technique (SIT), Basic reproduction number, Lymphatic Filariasis infection.

I. INTRODUCTION

Lymphatic filariasis (LF) is a mosquito-borne parasitic disease caused by thread-like parasitic worms such as *Wuchereria bancrofti*, *Brugia malayi,* and *Brugia timori*. The infection spreads from person to person by mosquito bite in the process of taking blood meals [1, 2].Transmission control can be done through annual Mass Drug Administration (MDA) (i.e. a combination of two drugs *albendazole* and *diethyl-carbamazine* or *albendazole* and *ivermectin*) and vector control by using insecticide-treated bed nets. MDA reduces the intensity of blood *microfilaemia* (*Mf*) and vector control reduces the vector population or the vector biting rate. The parasite population can be controlled through selective or mass treatment. However, the most applicable control strategy in the world is the annual MDA [1]. Selective treatment is expensive and cumbersome as it involves detection of all *Mf* carriers and night blood screening of entire population using invasive blood sampling procedures [3]. In mass treatment all persons in the community, irrespective of their Mf-status, are treated. The continuing prevalence of LF, possibility of drug resistance of parasite against MDA and insecticides used against mosquitoes [4, 5] , calls for an urgent need to develop an alternative strategy to supplement MDA to achieve LF elimination.

To encounter the situation, this study intends to integrate vector management with MDA through health education campaign and Sterile Insect Technique to tackle the disease. Health education campaigns empower community members, on the use of mosquito insecticide treated nets (ITN), indoor residual spraying (IRS) and environmental cleanliness. It is important to create awareness and change attitude of community members in endemic areas with regard to vector control. For example knowledge on the essence of using ITN, personal protection, treating septic tanks , eliminating breeding sites and inverting all containers holding water in the house premises in

many towns and villages especially in rainy seasons. SIT is a biological control in which the natural reproductive process of insects is disrupted by the use of mutagens such as gamma radiation thus rendering the insects sterile [6]. These sterile insects are then released into the environment in order to mate with the native insects that are present in the environment. SIT reduces vector abundance since females that mate with sterile males produce unviable eggs of which the native insect population should decrease. Several mathematical models have been developed to study the parasite transmission and control of LF [7, 8, 9, 10, 5, 11, and 12]. In particular, [7] developed a deterministic model to investigate the dynamics of LF disease through mosquitoes in four stages of human population based on the microfilariae and antibody levels. The stages are functions of transmission rates, reproduction, mortality, drug chemotherapy, bed net usage and insecticide spraying. The result indicated that a combination of MDA, bed-nets and spraying is the best solution in controlling the disease. [12] developed a LYMFASIM simulation models for predicting the impact of lymphatic Filariasis control. The model represented the relationship between mosquito biting rate and prevalence of microfilariae (mf) in the human population. The study suggested that, the impact of mass treatment depends strongly on the mosquito biting rate, the assumed coverage, compliance and drug efficacy. [5], developed a mathematical model of LF transmission and controls. The study revealed that the effect of combining vector control with drug administration has a greater impact on reducing infection levels in the community for each year of control, but also the target infection level is reached faster and earlier compared with using drug regime only. [13] used computer simulation modeling (LYMFASIM) to study how increasing MDA frequency from once to twice per year would affect program duration to eliminate LF for Indians and West African. The model prediction suggested that semiannual MDA is likely to reduce the time required to eliminate LF by 50% in most situations. [10] developed a model to quantify the potential effect that heterogeneous infection processes occurring in the major mosquito vector genera may have on parasite transmission and control. The findings indicated that filarial infection thresholds, system resilience, transmission dynamics and parasite response to control efforts, can all be influenced by the prevailing transmitting mosquito genus. [9] used mathematical models to assess the feasibility and strategic value of including vector control in the GPELF initiative to achieve the global elimination of LF. The findings indicated three major strategic roles for including vector controlling LF elimination programs; first, transmission elimination will be accelerated by raising worm breakpoint thresholds and by reducing the number of years of required drug intervention. Second, the drug coverage required will be lowered. Third, long-term parasite elimination from treated communities will be sustained by raising the infection thresholds to prevent the re-emergence of stable transmission. However, none of these studies, considered the role of MDA, health education campaign to community (to comply with ITN, IRS and environmental cleanliness) and Sterile Insect Technique (SIT). The model differs from previous works since no mathematical study has been conducted to establish the effects of multi-intervention campaigns on transmission dynamic of LF disease.

II. MODEL FORMULATION

The model sub-divides the total human population at time t, denoted by $N_h(t)$, into the following sub-populations of susceptible individuals $S_h(t)$: educated individuals $A(t)$, exposed individuals (infected but not infectious) $E_h(t)$, individuals with Lymphatic Filariasis symptoms (infectious) $I_h(t)$ and recovered individuals with temporary immunity $R(t)$.The total population is governed by

$$N_h(t) = S_h(t) + A(t) + E_h(t) + I_h(t) + R(t) \quad (1)$$

The total vector (mosquito) population at time t, denoted by $N_v(t)$, is sub-divided into susceptible mosquitoes $S_v(t)$, exposed mosquitoes $E_v(t)$ and infectious mosquitoes $I_v(t)$. Thus,

$$N_v(t) = S_v(t) + S_s(t) + E_v(t) + I_v(t) \quad (2)$$

The susceptible humans S_h are recruited into the population by birth and immigrants (infected immigrants are not included because it is assumed that most people who are sick will not travel) at a constant rate Λ and recovery individuals from R at a rate of ω. Susceptible individuals are given preventive chemotherapy Mass Drug Administration (MDA) and progress to education class A at a rate of ϕ (public health education is given on the use of Insecticides Treated Nets (ITN), Indoor Residual Spraying (IRS) and environmental cleanliness). It is assumed that those who received mass prevention are educated to comply with MDA. Φ is defined as the reduction of likelihood of infection by health education. The likelihood of infection is assumed to be reduced by a factor Φ. This means that health education is effective if $\Phi = 0$ and ineffective if $\Phi = 1$; Health education will be effective if there would be no progression of individuals from educated class to exposed class when $\Phi = 0$.

Susceptible human S_h acquire LF through contact with infected mosquito at a rate λ_h and move to exposed class. Infected human individuals acquire infections from exposed class at a rate ν and progress to the recovery state R at a treatment rate ε. Since the disease has temporary immunity, the recovered individuals lose their immunity at the rate of ω and return to the susceptible class. All human populations are subject to constant natural death at a rate of μ_h. It is assumed that the disease is not fatal, so there is no death associated with the disease. Susceptible mosquitoes are generated at the rate π.

Susceptible female mosquitoes mating with non-sterile male mosquitoes S_v and susceptible female mosquitoes mating with sterile male mosquitoes S_s are generated at rate the π. Female mosquitoes mating with sterile male mosquitos' progress to S_s at a proportion of ρ while susceptible female mosquitoes mating with non-sterile male mosquitoes' moves to S_v class at the other proportion $(1-\rho)$. Susceptible female mosquitoes mating with non-sterile male mosquitoes' S_v acquire LF through contact with infected human at a rate λ_v and move to exposed class. We assume that sterilized male mosquitos' stops reproduction completely. However, susceptible female mosquitoes S_s mating with sterile male mosquitoes acquire infections through contact with infected human and progress to exposed class at $\psi\lambda_v$. That is, the S_s mosquitos' population remains constant which leads to reduction of disease transmission. ψ is defined as the reduction of likelihood of infection by sterile male mosquitoes.

Finally, exposed mosquito progress to infectious class I_v at the rate of γ. Deaths occur at a rate μ_v in each of the mosquito classes. It is assumed that $\lambda_h = \frac{\alpha\beta_{vh}I_v}{N_v}$, $\lambda_v = \frac{\alpha\beta_{hv}I_h}{N_h}$. As in [14] the terms $\frac{\alpha\beta_{vh}I_v}{N_v}$ denotes the rate at which the human hosts S_h get infected by infected mosquitoes I_v and $\frac{\alpha\beta_{hv}I_h}{N_h}$ refers to the rate at which the susceptible mosquitoes S_v and S_s are infected by the infected human hosts. β_{vh} is the probability that a human host becomes infected by infectious mosquitoes and β_{hv} is the probability that susceptible mosquitoes become infected by biting infected human. α refers to the mosquito biting rate.

The proposed model satisfies the assumption that host and vector populations are not constant, there is no vertical transmission meaning that all newly born are susceptible to infection, and we

assume that there is no natural recovery to human population. However, the model assumes that sterile male mosquitoes are released in sufficient numbers over a sufficient period of time.

Taking into account the above considerations and assumptions, we then have the following schematic flow diagram:

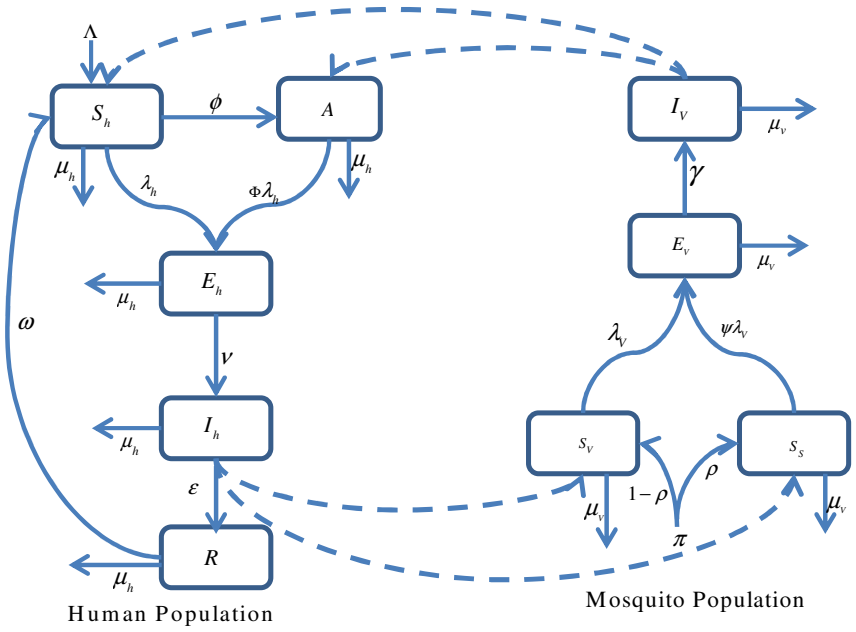

Figure 1: The Lymphatic Filariasis model with interventions

The above flow diagram leads to the system of ordinary differential equations which describes the dynamics of the disease as follows:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda + \omega R - \lambda_h S_h - (\phi + \mu_h) S_h \\
\frac{dA}{dt} &= \phi S_h - (\Phi\lambda_h + \mu_h) A \\
\frac{dE_h}{dt} &= \lambda_h S_h + \Phi\lambda_h A - (v + \mu_h) E_h \\
\frac{dI_h}{dt} &= vE_h - (\varepsilon + \mu_h) I_h \\
\frac{dR}{dt} &= \varepsilon I_h - (\omega + \mu_h) R \\
\frac{dS_v}{dt} &= (1-\rho)\pi - (\lambda_v + \mu_v) S_v \\
\frac{dS_s}{dt} &= \rho\pi - (\psi\lambda_v + \mu_v) S_s \\
\frac{dE_v}{dt} &= \lambda_v S_v + \psi\lambda_v S_s - (\gamma + \mu_v) E_v \\
\frac{dI_v}{dt} &= \gamma E_v - \mu_v I_v
\end{aligned} \tag{3}$$

III. MODEL ANALYSIS

3.1. Positivity of solutions

For the model (3) to be epidemiologically meaningful and well posed, we need to prove that all state variables are non-negative $\forall t \geq 0$.

Lemma 1

Let $\{(S_h(0), S_v(0), S_s(0)) > 0, (E_h(0), I_h(0), R(0), E_v(0), I_v(0) \geq 0)\} \in \Gamma$. Then the solution $\{S_h, A, E_h, I_h, R, S_v, S_v, E_v, I_v\}(t)$ of the model system (3) is positive for all $t \geq 0$.

Proof

From the first equation of system (3), we have

$\frac{dS_h}{dt} = \Lambda - \lambda_h S_h + \omega R - \mu_h S_h \geq -(\lambda_h + \phi + \mu_h) S_h$ this upon integration gives

$S_h(t) \geq S_h(0) e^{-(\int \lambda_h + \phi + \mu_h) dt} \geq 0$ as $t > \infty$, we obtain

$S_h(t) \geq S_h(0)$

Similarly, it can be shown that all the remaining state variables of the system (3), are also positive for all $t > 0$. Hence, all feasible solution of system (3), enters the invariant region

$\Gamma = \{S_h, A, E_h, I_h, R, S_v, S_s, E_v, I_v\}$.

3.2. Steady state solutions

Setting the right hand side of system (3) to zero in terms of λ_h^* and λ_v^* we obtain the following

$$S_h^* = \frac{\Lambda + \omega R^*}{\lambda_h + (\phi + \mu_h)} \qquad A^* = \frac{\phi S_h^*}{\Phi \lambda_h + \mu_h} \qquad E_h^* = \frac{\left(S_h^* + \Phi A^*\right)\lambda_h^*}{(\nu + \mu_h)}$$

$$I_h^* = \frac{\nu E_h^*}{(\varepsilon + \mu_h)} \qquad R^* = \frac{\varepsilon I_h^*}{(\omega + \mu_h)} \qquad S_v^* = \frac{(1-\rho)\pi}{(\lambda_v^* + \mu_v)}$$

$$S_s^* = \frac{\rho\pi}{(\lambda_v^* + \mu_v)} \qquad E_v^* = \frac{(S_v^* + \psi S_s^*)\lambda_v^*}{(\gamma + \mu_v)} \qquad I_v^* == \frac{\gamma(S_v^* + \psi S_s^*)\lambda_v^*}{\mu_v(\gamma + \mu_v)}$$

Substituting I_v^* and I_h^* in the expression for the forces of infection λ_v^* and λ_h^* respectively, we obtain

$$\lambda_h = \alpha\beta_{vh} \frac{1}{\mu_v} \frac{\gamma}{(\gamma + \mu_v)} \left(\frac{S_v^* + \psi S_s^*}{N_v}\right) \lambda_v^* \tag{4}$$

$$\lambda_v = \alpha\beta_{hv} \frac{\nu}{(\nu + \mu_h)} \frac{1}{(\varepsilon + \mu_h)} \left(\frac{S_h^* + \Phi A}{N_h}\right) \lambda_h^* \tag{5}$$

Upon substitution of λ_h^* into λ_v^* we obtain

$$\lambda_v^* = \frac{\alpha^2 \beta_{vh} \beta_{hv} \nu\gamma}{(\nu + \mu_h)(\gamma + \mu_v)\mu_v} \frac{1}{(\varepsilon + \mu_h)} \left(\frac{S_h^* + \Phi A}{N_h}\right) \left(\frac{S_v^* + \psi S_s^*}{N_v}\right) \lambda_v^* \tag{6}$$

This gives

$$\lambda_v^* = 0 \tag{7}$$

or

$$\frac{\alpha^2 \beta_{vh} \beta_{hv} \nu\gamma}{(\nu+\mu_h)(\gamma+\mu_v)\mu_v} \frac{1}{(\varepsilon+\mu_h)} \left(\frac{S_h^* + \Phi A}{N_h}\right)\left(\frac{S_v^* + \psi S_s^*}{N_v}\right) = 1 \tag{8}$$

3.2.1. Disease free equilibrium point

The solution $\lambda_v^* = 0$ leads to the disease free equilibrium point given by

$$E_0 = \left(\frac{\Lambda}{\phi+\mu_h}, \frac{\Lambda\phi}{\mu_h(\phi+\mu_h)}, 0, 0, 0, \frac{(1-\rho)\pi}{\mu_v}, \frac{\rho\pi}{\mu_v}, 0, 0\right)$$

3.2.2. Existence of endemic equilibrium point.

The solution $\frac{\alpha^2 \beta_{vh} \beta_{hv} \nu\gamma}{(\nu+\mu_h)(\gamma+\mu_v)\mu_v} \frac{1}{(\varepsilon+\mu_h)} \left(\frac{S_h^* + \Phi A}{N_h}\right)\left(\frac{S_v^* + \psi S_s^*}{N_v}\right) = 1$ of (8) leads to the endemic equilibrium point given by $E_1 = \left(S_h^*, A^*, E_h^*, I_h^*, R^*, S_v^*, S_s^*, E_v^*, I_v^*\right)$

$$S_h^* = \frac{\Lambda + \omega R^*}{\lambda_h + (\phi + \mu_h)} \qquad A^* = \frac{\phi S_h^*}{\Phi\lambda_h + \mu_h} \qquad E_h^* = \frac{\left(S_h^* + \Phi A^*\right)\lambda_h^*}{(\nu + \mu_h)}$$

$$I_h^* = \frac{\nu E_h^*}{(\varepsilon+\mu_h)} \qquad R^* = \frac{\varepsilon I_h^*}{(\omega+\mu_h)} \qquad S_v^* = \frac{(1-\rho)\pi}{(\lambda_v^* + \mu_v)}$$

$$S_s^* = \frac{\rho\pi}{(\lambda_v^* + \mu_v)} \qquad E_v^* = \frac{(S_v^* + \psi S_s^*)\lambda_v^*}{(\gamma + \mu_v)} \qquad I_v^* == \frac{\gamma(S_v^* + \psi S_s^*)\lambda_v^*}{\mu_v(\gamma + \mu_v)}$$

3.3. Basic reproduction number R_e

The basic reproduction number or contact number R_e represents the average number of secondary infections that a single infection host can generate in a totally susceptible population of hosts and vectors. We calculate the basic reproduction number R_e by using the next generation method by [15] of the system (3).

$$A = \left[\frac{\partial F_i(E_0)}{\partial x_i}\right] \cdot \left[\frac{\partial V_i(E_0)}{\partial x_i}\right]^{-1} = FV^{-1} \tag{9}$$

where

F_i is the rate of appearance of new infections and $V_i = V_i^- - V_i^+$ is the net rate of transfer of individuals into compartment i, with V_i^- denoting transfer out of compartment i and V_i^+ the transfer of individuals into compartment i. This model has four (4) infected classes, thus $m = 4$; and are ordered as follows: E_h, I_h, E_v, and I_v. The matrices F and V are obtained from model (3) as:-

The Jacobian matrix of **F** evaluated at E_0 is given by

$$\mathbf{F}=\begin{bmatrix} 0 & 0 & 0 & \dfrac{\beta_{vh}\alpha\Lambda\mu_v(\phi\Phi+\mu_h)}{\pi\mu_h(\phi+\mu_h)} \\ 0 & 0 & 0 & 0 \\ 0 & \dfrac{\beta_{hv}\alpha\pi\mu_h((1-\rho)+\psi\rho)}{\Lambda\mu_v} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The Jacobian matrix of $\mathbf{V}$ evaluated at E_0 is given by

$$\mathbf{V}=\begin{bmatrix} (\nu+\mu_v) & 0 & 0 & 0 \\ -\nu & (\varepsilon+\mu_h) & 0 & 0 \\ 0 & 0 & (\gamma+\mu_v) & 0 \\ 0 & 0 & -\gamma & \mu_v \end{bmatrix}$$

R_e is defined as the spectral radius (dominant eigenvalues) of the matrix $\mathbf{FV}^{-1}$ which is

$$R_e=\sqrt{\frac{\alpha^2\beta_{vh}\beta_{hv}\nu\gamma(\phi\Phi+\mu_h)(1-\rho+\psi\rho)}{(\phi+\mu_h)(\nu+\mu_h)(\varepsilon+\mu_h)(\gamma+\mu_v)\mu_v}}=\sqrt{R_{e_h}.R_{e_v}} \qquad (10)$$

where

$$R_{e_h}=\sqrt{\frac{\alpha\beta_{vh}\nu(\phi\Phi+\mu_h)}{(\phi+\mu_h)(\nu+\mu_h)(\varepsilon+\mu_h)}}$$

and $$R_{e_v}=\sqrt{\frac{\alpha\beta_{hv}\gamma(1-\rho+\psi\rho)}{(\gamma+\mu_v)\mu_v}}$$

The basic reproduction number R_e is used to determine whether the disease becomes persistent or dies out depending on magnitude of R_e. Stability of the equilibrium points can be analysed using R_e.

3.4. Local Stability of Disease Free Equilibrium Point E_0

Theorem 1:

The disease free equilibrium point is locally asymptotically stable if and only if $R_e<1$, *and is unstable if* $R_e>1$.

The Jacobian matrix of the system (3) evaluated at E_0 is given by

$$J_{E_0}=\begin{bmatrix} -P & 0 & 0 & 0 & \omega & 0 & 0 & 0 & H \\ \phi & -\mu_h & 0 & 0 & 0 & 0 & 0 & 0 & -J \\ 0 & 0 & -\sigma & 0 & 0 & 0 & 0 & 0 & \Delta \\ 0 & 0 & \nu & -\partial & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & -\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\ell & 0 & -\mu_v & 0 & 0 & 0 \\ 0 & 0 & 0 & -W & 0 & 0 & -\mu_v & 0 & 0 \\ 0 & 0 & 0 & \kappa & 0 & 0 & 0 & -\xi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & -\mu_v \end{bmatrix} \qquad (11)$$

where $P=(\phi+\mu_h)$ $H=\dfrac{\alpha\beta_{vh}\Lambda\mu_v}{(\phi+\mu_h)\pi}$ $J=\dfrac{\Phi\phi\alpha\beta_{vh}\Lambda\mu_v}{\pi\mu_h(\phi+\mu_h)}$ $\sigma=(\nu+\mu_h)$,

$\Delta=\dfrac{\alpha\beta_{vh}\Lambda\mu_v(\Phi\phi+\mu_h)}{\pi\mu_h(\phi+\mu_h)}$ $\partial=(\varepsilon+\mu_h)$ $\delta=(\omega+\mu_h)$ $\xi=(\gamma+\mu_v)$,

$\ell=\dfrac{\alpha\beta_{hv}(1-\rho)\pi\mu_h}{\Lambda\mu_v}$ $W=\dfrac{\psi\alpha\beta_{hv}\rho\pi\mu_h}{\Lambda\mu_v}$, $\kappa=\dfrac{\alpha\beta_{hv}\pi\mu_h(1-\rho+\psi\rho)}{\Lambda\mu_v}$

The characteristic polynomial of J_{E_0} is given by $\left|J_{E_0}-\lambda I\right|=0$ giving rise to $\lambda_1=-\mu_h$, $\lambda_2=-(\phi+\mu_h)$, $\lambda_3=-(v+\mu_h)$, $\lambda_4=-(\varepsilon+\mu_h)$, $\lambda_5=-(\omega+\mu_h)$, $\lambda_6=-\mu_v$, $\lambda_7=-\mu_v$. $\lambda_8=-(\gamma+\mu_v)$, $\lambda_9=-\mu_v$. Since $\lambda_1<\lambda_2<...<\lambda_9<0$ (both eigenvalues are real and negative) then the equilibrium is *locally asymptotically stable.*

3.5. Global stability of the disease free equilibrium point E_0

Theorem 2.

The disease-free equilibrium point is E_1 *globally asymptotically stable if* $R_e<1$ *and unstable if* $R_e>1$.

In this section, we analyse the global stability of the disease-free equilibrium point by applying the [16] approach. We write model system (3) in the form:

$$\frac{d\mathbf{X}_s}{dt}=\mathbf{A}\left(X_s-X_{DFE}\right)+\mathbf{A}_1X_i$$
$$\frac{d\mathbf{X}_i}{dt}=\mathbf{A}_2X_i \qquad (12)$$

where,

$\mathbf{X}_s$ is the vector representing the non-transmitting compartments and $\mathbf{X}_i$ is the vector representing the transmitting components. The DFE is globally asymptotically stable if $\mathbf{A}$ has real negative eigenvalues and $\mathbf{A}_2$ is a Metzler matrix (i.e. the off-diagonal elements of $\mathbf{A}_2$ are non-negative). A Metzler matrix $\mathbf{A}$ is a matrix whose off diagonal elements are non- negative such that $\mathbf{A}(i,j)>0$ for any indices $i\neq j$.

From system (3) we have, $\mathbf{X}_i=\left(E_h,I_h,E_v,I_v\right)^T$ and $\mathbf{X}_s=\left(S_h,A,R,S_v,S_s\right)^T$

$$\mathbf{X}_s-\mathbf{X}_{DFE}=\begin{bmatrix}S_h\\A\\R\\S_v\\S_s\end{bmatrix}-\begin{bmatrix}S_h-\dfrac{\Lambda}{(\phi+\mu_h)}\\A-\dfrac{\phi\Lambda}{(\phi+\mu_h)\mu_h}\\R\\S_v-\dfrac{(1-\rho)\pi}{\mu_v}\\S_s-\dfrac{\rho\pi}{\mu_v}\end{bmatrix} \qquad (13)$$

We need to check whether a matrix $\mathbf{A}$ for the non-transmitting compartments has real negative eigenvalues and that $\mathbf{A}_2$ is a Metzler matrix. From the equation for non-transmitting compartments in (3) we have:

$$\mathbf{A}=\begin{bmatrix} -(\phi+\mu_h) & 0 & \omega & 0 & 0 \\ \phi & -\mu_h & 0 & 0 & 0 \\ 0 & 0 & -(\omega+\mu_h) & 0 & 0 \\ 0 & 0 & 0 & -\mu_v & 0 \\ 0 & 0 & 0 & 0 & -\mu_v \end{bmatrix} \tag{14}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & -\frac{\beta_{vh}\alpha S_h}{N_v} \\ 0 & 0 & 0 & -\frac{\Phi\beta_{vh}\alpha S_h}{N_v} \\ 0 & \varepsilon & 0 & 0 \\ 0 & -\frac{\alpha\beta_{hv}S_v}{N_h} & 0 & 0 \\ 0 & -\frac{\psi\alpha\beta_{hv}S_s}{N_h} & 0 & 0 \end{bmatrix} \tag{15}$$

and $$\mathbf{A}_2=\begin{bmatrix} -(\nu+\mu_h) & 0 & 0 & \frac{\alpha\beta_{vh}(S_h+\Phi A)}{N_v} \\ \nu & -(\varepsilon+\mu_h) & 0 & 0 \\ 0 & \frac{\alpha\beta_{hv}(S_v+\psi S_s)}{N_h} & -(\gamma+\mu_v) & 0 \\ 0 & 0 & \gamma & -\mu_v \end{bmatrix} \tag{16}$$

A direct computation shows that, the eigenvalues of $\mathbf{A}$ are real and negative. This implies that the system $\frac{d\mathbf{X}_s}{dt}=\mathbf{A}\left(X_s-X_{DFE}\right)+\mathbf{A}_1X_i$ is globally asymptotically stable at DFE.

Furthermore, to check the stability of the system $\mathbf{A}_2(x)$ must be Metzler stable matrix.
Thus, the approach by [17] and [18] will be employed to check the stability.

Lemma 2

Let $\mathbf{V}$ be a square Metzler matrix written in block form

$$\mathbf{V}=\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$$

with $\mathbf{A}$ and $\mathbf{D}$ are square matrices. $\mathbf{V}$ is Metzler stable if and only if the matrices $\mathbf{A}$ and $\mathbf{D}$-$\mathbf{CA}^{-1}\mathbf{B}$ are Metzler stable.

Proof.

Comparing the matrix $\mathbf{A}_2$ and a square Metzler matrix $\mathbf{V}$ given, we have matrices $\mathbf{A,B,C}$ and $\mathbf{D}$ defined as;

$$\mathbf{A}=\begin{pmatrix} -(\nu+\mu_h) & 0 \\ \nu & -(\varepsilon+\mu_h) \end{pmatrix} \qquad \mathbf{B}=\begin{pmatrix} 0 & \frac{\alpha\beta_{vh}\Lambda\mu_v(\Phi\phi+\mu_h)}{\pi\mu_h(\phi+\mu_h)} \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{C}=\begin{pmatrix} 0 & \dfrac{\alpha\beta_{hv}\pi\mu_h(1-\rho+\psi\rho)}{\Lambda\mu_v} \\ 0 & 0 \end{pmatrix} \qquad \mathbf{D}=\begin{pmatrix} -(\gamma+\mu_v) & 0 \\ \gamma & -\mu_v \end{pmatrix}$$

$$\mathbf{D\text{-}CA^{-1}B}=\begin{pmatrix} -(\gamma+\mu_v) & \dfrac{\alpha^2\beta_{vh}\beta_{hv}\nu(\Phi\phi+\mu_h)(1-\rho+\psi\rho)}{(\nu+\mu_h)(\varepsilon+\mu_h)(\phi+\mu_h)} \\ \gamma & -\mu_v \end{pmatrix}$$

Thus, $\mathbf{D\text{-}CA^{-1}B}$ is Metzler stable iff $\dfrac{\alpha^2\beta_{vh}\beta_{hv}\nu\gamma(\Phi\phi+\mu_h)(1-\rho+\psi\rho)}{(\nu+\mu_h)(\varepsilon+\mu_h)(\phi+\mu_h)}<(\gamma+\mu_v)\mu_v$

That is, $\dfrac{\alpha^2\beta_{vh}\beta_{hv}\nu\gamma(\Phi\phi+\mu_h)(1-\rho+\psi\rho)}{(\phi+\mu_h)(\nu+\mu_h)(\varepsilon+\mu_h)(\gamma+\mu_v)\mu_v}-1<0 \Rightarrow \dfrac{\alpha^2\beta_{vh}\beta_{hv}\nu\gamma(\Phi\phi+\mu_h)(1-\rho+\psi\rho)}{(\phi+\mu_h)(\nu+\mu_h)(\varepsilon+\mu_h)(\gamma+\mu_v)\mu_v}<1$

Therefore, $R_e^2<1 \Rightarrow R_e<1$

The disease-free equilibrium point is E_2 globally asymptotically stable if $R_e<1$ and unstable if $R_e>1$.

3.6. Global Stability of the endemic equilibrium point E_1

In this section, we analyze the global stability of the endemic equilibrium point by proving the theorem below:

Theorem 3.
The endemic equilibrium point, E_1 is globally asymptotically stable If $R_e>1$ and unstable if $R_e<1$.

Proof
In proving this theorem, Lyapunov function can be used to analyze the global stability of the endemic equilibrium using the approach of [19][20][21]. In this approach, we construct Lyapunov functions of the form,

$$\mathbf{L}=\sum a_i\left(\mathbf{X}_i-\mathbf{X}_i^*\ln X_i\right) \tag{17}$$

where a_i is properly selected constant, X_i is the population of the i^{th} compartment, and X_i^* is the equilibrium point. The approach has been found to be useful for compartmental epidemic models with any number of compartments. Thus, consider the Lyapunov function of the form

$$\begin{aligned} L=&a_1\left(S_h-S_h^*\ln S_h\right)+a_2\left(A-A^*\ln A\right)+a_3\left(E_h-E_h^*\ln E_h\right)+a_4\left(I_h-I_h^*\ln E_h\right)+a_5\left(R-R^*\ln R\right)\\ &+a_6\left(S_v-S_v^*\ln S_v\right)+a_7\left(S_s-S_s^*\ln S_s\right)+a_8\left(E_v-E_v^*\ln E_v\right)+a_9\left(I_v-I_v^*\ln I_v\right)\end{aligned} \tag{18}$$

where, $a_1,a_2,a_3,...,a_9$ are positive constants.

Differentiate the Lyapunov function with respect to time to get,

$$\begin{aligned}\frac{dL}{dt}=&a_1\left(1-\frac{S_h^*}{S_h}\right)\frac{dS_h}{dt}+a_2\left(1-\frac{A^*}{A}\right)\frac{dA}{dt}+a_3\left(1-\frac{E_h^*}{E_h}\right)\frac{dE_h}{dt}+a_4\left(1-\frac{I_h^*}{I_h}\right)\frac{dI_h}{dt}+a_5\left(1-\frac{R^*}{R}\right)\frac{dR}{dt}\\&+a_6\left(1-\frac{S_v^*}{S_v}\right)\frac{dS_v}{dt}+a_7\left(1-\frac{S_s^*}{S_s}\right)\frac{dS_s}{dt}+a_8\left(1-\frac{E_v^*}{E_v}\right)\frac{dE_v}{dt}+a_9\left(1-\frac{I_v^*}{I_v}\right)\frac{dI_v}{dt}\end{aligned}$$

From the system, and some computations we have,

$$\begin{aligned}\frac{dL}{dt}=&-a_1\left(1-\frac{S_h^*}{S_h}\right)^2(\phi+\mu_h)S_h-a_2\left(1-\frac{A^*}{A}\right)^2\mu_h-a_6\left(1-\frac{S_v^*}{S_v}\right)^2(\theta+\mu_v)S_v-a_7\left(1-\frac{S_s^*}{S_s}\right)^2\mu_vS_s\\&-a_1\left(1-\frac{S_h^*}{S_h}\right)\left(\beta_{vh}\alpha\mu_vI_vS_h-\left(R-R^*\right)\omega\right)-a_2\left(\left(1-\frac{I_v^*A^*}{I_vA}\right)\varphi\beta_{vh}\alpha\mu_vI_vA-\left(1-\frac{A^*}{A}\right)\left(1-\frac{S_h^*}{S_h}\right)\phi\right)\\&+a_3\left(1-\frac{E_h^*}{E_h}\right)\left(\left(1-\frac{I_v^*S_h^*E_h}{I_vS_hE_h^*}\right)\frac{\beta_{vh}\alpha\mu_v}{\pi}+\left(1-\frac{I_v^*A^*E_h}{I_vAE_h^*}\right)\frac{\varphi\beta_{vh}\alpha\mu_v}{\pi}\right)\\&+a_4\left(1-\frac{I_h^*}{I_h}\right)\left(1-\frac{E_h^*I_h}{E_hI_h^*}\right)\nu E_h-a_5\left(1-\frac{R^*}{R}\right)\left(\left(1-\frac{E_h^*R}{E_hR^*}\right)\delta E_h-\left(1-\frac{I_h^*R}{I_hR^*}\right)\varepsilon I_h\right)\\&-a_6\left(1-\frac{I_h^*S_v^*}{I_hS_v}\right)\frac{\beta_{hv}\alpha\mu_hI_hS_v}{\Lambda}-a_7\left(1-\frac{S_s^*}{S_s}\right)\left(\left(1-\frac{I_h^*S_S^*}{I_hS_S}\right)\frac{\psi\beta_{hv}\alpha\mu_hI_hS_s}{\Lambda}-\left(1-\frac{S_v^*}{S_v}\right)\theta S_v\right)\\&+a_8\left(1-\frac{E_v^*}{E_v}\right)\left(\left(1-\frac{I_h^*S_v^*E_v}{I_hS_vE_v^*}\right)\frac{\beta_{hv}\alpha\mu_h}{\Lambda}+\left(1-\frac{I_h^*S_s^*E_v}{I_hS_sE_v^*}\right)\frac{\psi\beta_{hv}\alpha\mu_h}{\Lambda}\right)+a_9\left(1-\frac{I_v^*}{I_v}\right)\left(1-\frac{E_v^*I_v}{E_vI_v^*}\right)\gamma E_v\end{aligned}\tag{19}$$

This can be written as:

$$\frac{dL}{dt}=-a_1\left(1-\frac{S_h^*}{S_h}\right)^2(\phi+\mu_h)S_h-a_2\left(1-\frac{A^*}{A}\right)^2\mu_h-a_6\left(1-\frac{S_v^*}{S_v}\right)^2(\theta+\mu_v)S_v-a_7\left(1-\frac{S_s^*}{S_s}\right)^2\mu_vS_s+F(\Re)\tag{20}$$

where, $F(\Re)$ is the balance of the right hand terms of (20) .

Following the approach of [19][22][23], F is non-positive for $S_h,A,E_h,I_h,R,S_v,S_S,E_v,I_v>0$. Therefore,

$\frac{dL}{dt}=0$ if $S_h=S_h^*,\ A=A^*,\ E_h=E_h^*,\ I_h=I_h^*,\ R=R^*,\ S_v=S_v^*,\ S_S=S_s^*,\ E_v=E_v^*,\ I_v=I_v^*$

$\frac{dL}{dt}<0$ for $S_h,A,E_h,I_h,R,S_v,S_S,E_v,I_v>0$.

Thus, if $R_{eff}>1$ then, model system (3) has endemic equilibrium point E_1 which is *globally asymptotically stable*.

IV. SENSITIVITY ANALYSIS AND NUMERICAL SIMULATIONS

4.1. Sensitivity analysis

Numerical sensitivity analysis was done by computing sensitivity indices of reproduction number R_e which measures initial disease transmission using the approach by [24]. Sensitivity analysis determines parameters that have a high impact on R_e and should be targeted by intervention strategies. We derive an analytical expression for the sensitivity of R_e as $r_q^{R_e}=\frac{\partial R_e}{\partial q}\times\frac{q}{R_e}$, to each parameters involved in R_e . For example sensitivity index of R_e with respect to α is $r_\alpha^{R_e}=\frac{\partial R_e}{\partial\alpha}\times\frac{\alpha}{R_e}=1$, $r_\gamma^{R_e}=\frac{\partial R_e}{\partial\gamma}\times\frac{\gamma}{R_e}=0.4091$. Other indices are obtained in the similar way and the results are displayed in table 1 below.

Table 1 Sensitivity indices of R_e

Parameter	Sensitivity Index
μ_v	-1.9091

α	1.0000
ε	-0.8823
β_{hv}	0.5000
β_{vh}	0.5000
ϕ	-0.4815
γ	0.4091
μ_h	-0.3570
Φ	0.3518
ρ	-0.2994
ν	-0.2792
ψ	0.2602

4.2. Interpretation of sensitivity indices

From table1 above, it shows that the following parameters α , β_{vh} , β_{hv} , Φ , γ and ψ have positive indices, meaning that when each of these parameters increases, they increases the value of R_e , implying that they increase the endemicity level of the disease. While the parameters μ_v , ε , ϕ , μ_h , ν and ρ have negative indices, implying that the increase of each of these parameters keeping other parameters constant lead to the decrease of R_e . This means that their increase, decreases the endemicity level of the disease in the community.

On the other hand, the most sensitive parameter is the mosquito mortality rate μ_v , the mosquito biting rate α , probability that a human host becomes infected by infectious mosquitoes and probability that mosquitoes become infected by infectious human. However, the progression rate ϕ after mass prevention to educated class and the progression rate γ of vectors from exposed class to infected class are important parameters. Human mortality rate μ_h and the reduction of likelihood of infection by health education Φ are another worth parameters.

Therefore, it can be interpreted that increasing the mosquitoes death rate and reducing mosquito biting rate (through education campaign on using bed nets, indoor spraying, environmental cleanliness and released sterile males' mosquitoes to lower infective female population) and increasing mass treatment would have the positive effect in controlling Lymphatic Filariasis transmission in the population.

4.3. Numerical simulation

In this section, we illustrate the analytical results of this work by carrying out numerical simulations of the model. This is done by using a set of parameter values whose sources are mainly from literature as well as assumptions as shown in table 2. The model systems are simulated using Matlab ODE solvers and the following initial conditions have been considered; the parameter values are given in table 2 and the values of the state variables are as shown.

$S_h = 10,000,\ A = 8000,\ E_h = 6000,\ I_h = 5000,\ R = 3000,\ S_v = 100,000,\quad S_s = 90,000,\ 60,000\ E_v = 60,000,.$
$I_v = 30,000$

Table 2 Model parameter values

Parameter	Parameter Value	Source
α	0.29	[25]
γ	0.0017	Assumed
β_{vh}	0.86	[26]

β_{hv}	0.283	Assumed
ν	1/17	[27]
ε	0.125	[28]
μ_h	0.0167	[29]
μ_v	0.017	[28]
π	0.071	[26]
Φ	0.475	[30]
ϕ	0.083	[30]
Λ	0.0384	[24]
ω	0.000137	Assumed
ρ	0.7	Assumed
ψ	0.465	Assumed

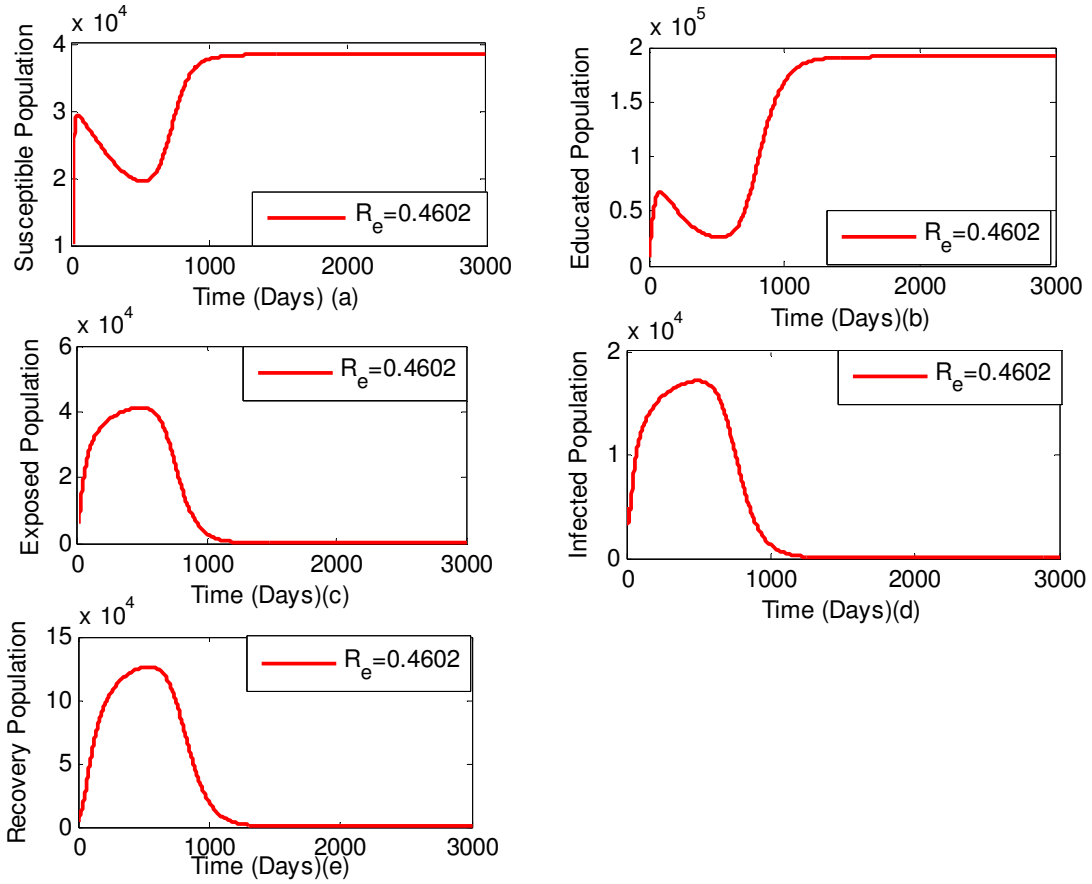

Figure 2: Dynamics of human populations of LF model with interventions

Figure 2 Illustrates the changes in the state variables of the LF model for $\mathrm{Re} = 0.4602$ (a) shows the dynamics susceptible individuals with time, (b) shows the dynamics of susceptible educated individuals with time, (c) shows the dynamics of exposed individuals who progress to infectious class with time, (d) shows the dynamics of infected (infectious) individuals with time, (e) Shows the dynamics of recovered individuals with time.

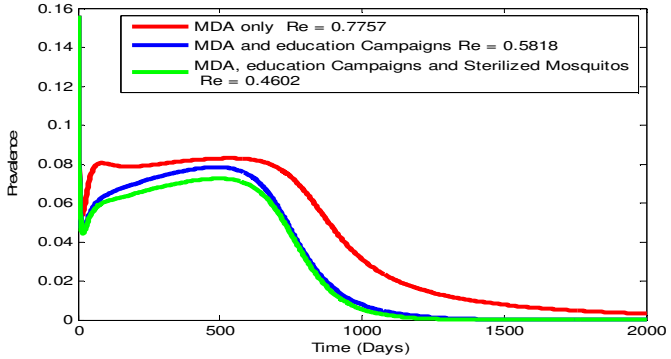

Figure 2: Changes of prevalence of the Lymphatic Filariasis model with interventions

Figure 2 shows the impact of combining all interventions (mass treatment, education campaign and sterilized mosquitoes) on the dynamics of disease prevalence. It seems that the combination of all interventions tend to reduce the disease prevalence in community than using Mass treatment (MDA) only. It is observed that when mass treatment only is used to control the disease, the reproduction number is larger $(\text{Re} = 0.7757)$ than when all interventions are combined where the reproduction number is $(\text{Re} = 0.4602)$. Therefore combined intervention may hasten the elimination of the disease compared to MDA only.

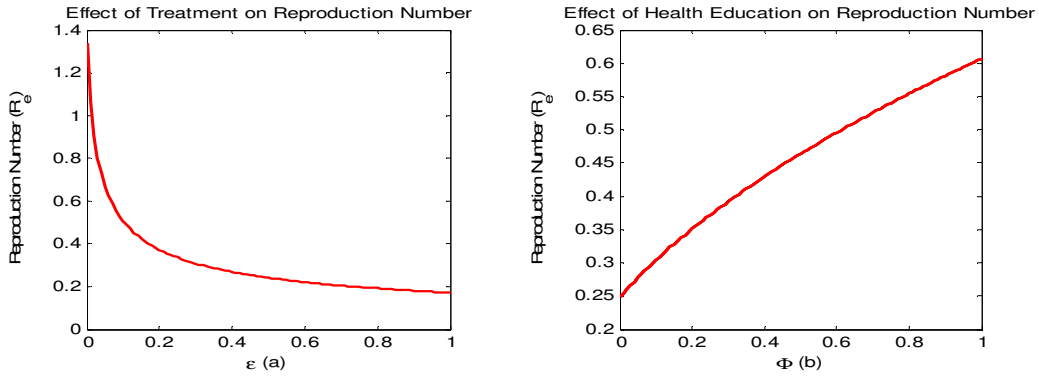

Figure 3: Shows the effect of some parameters on reproduction numbers

Figure 3(a): Shows the relationship between the reproduction numbers as ε changes. It is observed that when ε increases R_e reduces. While when health education is introduced, it is observed that R_e keeps on increasing implying that the small value of Φ the more effective the intervention is as shown in figure 3(b). Therefore, we can say that, when LF infected individuals are treated, health education on self-protection, the use of mosquito nets and indoor spraying should be conducted to reduce the levels of infection in the community.

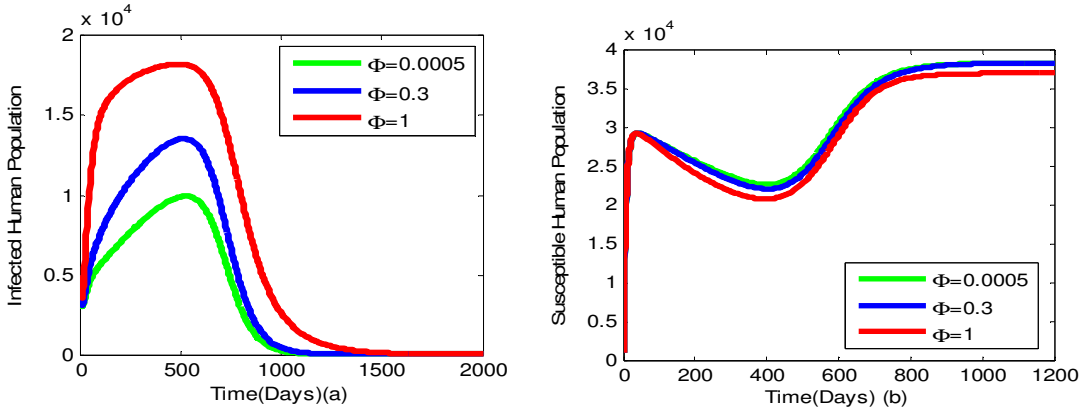

Figure 4: Variations of infected population and susceptibles population for different values of Φ.

It can be observed in figure 4(a), that the infected human population increases in the level of infection due to the increase of Φ. Again, figure 4(b) shows that the susceptible population reduces as Φ increase. This implies that the small the value of Φ is the more effective it is since this is the likelihood of reduction of infection due to education. Therefore small value of Φ shows few numbers of infected individuals. In figure 4(b), it is observed that the susceptible population reduces when there is an increase in Φ and vice versa.

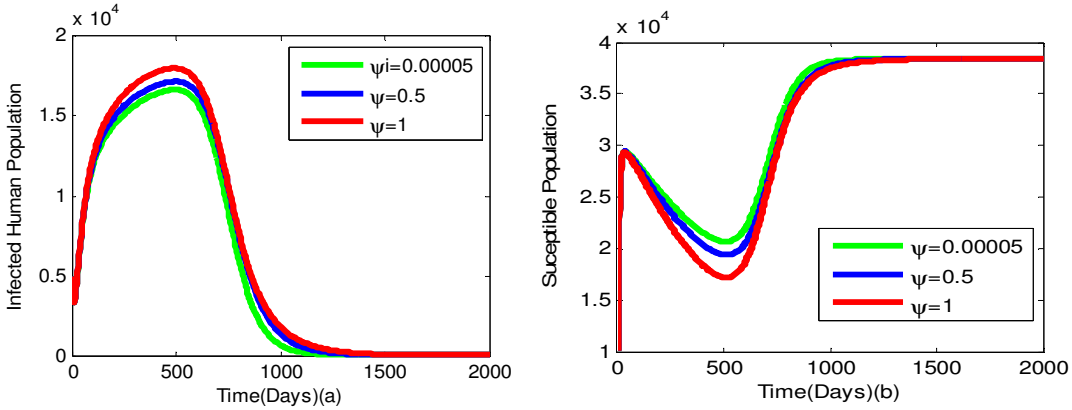

Figure 5: Variations of infected population and susceptibles population for different values of ψ

It can be observed in figure 5(a), that the infected human population decreases in the level of infection due to the decrease of ψ. Again, figure 5(b) shows that the susceptible population reduces as ψ increase. ψ is the reduction of likelihood of infection by sterile male mosquitoes. Therefore, when the value of ψ decreases, then the intervention is effective and that can be observed in figure 5(a) where small value of ψ, then few infected individuals. Likewise for Susceptible population that large value of ψ few susceptible individuals and vice versa.

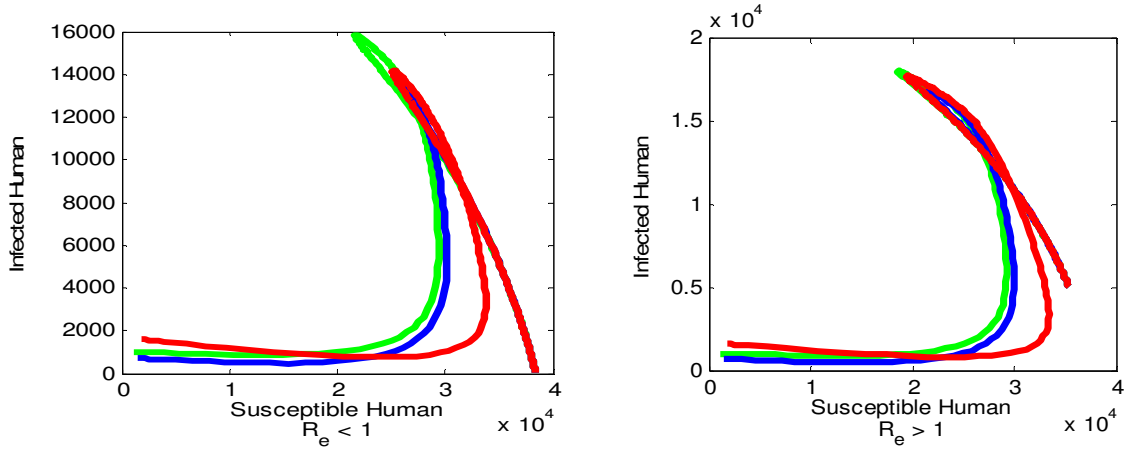

Figure 6: Phase portraits showing the projection of different classes

Figure 6(a) illustrates the phase portrait of dynamics of susceptible human and infected individuals. It is observed that the trajectory for any initial populations end up in a situation where there is no infected individuals, that is the disease free equilibrium point.

While in figure 6(b), it is observed that the trajectory for the initial populations end up in a situation where there is infected individuals that is, the endemic equilibrium point. Therefore these results confirm the stability results established.

V. DISCUSSION AND CONCLUSION

In this study, we develop and analyze a LF disease transmission mathematical model to determine the impact of multi-interventions like health education (on using Insecticides Treated Nets, Indoor Residual Spraying and environmental cleanliness) and Sterile Insect technique (SIT) on the spread of LF. We carried out a stability analysis of the equilibria. DFE and endemic equilibria are found to be locally and globally asymptotically stable. Sensitivity analysis of the key parameters and numerical simulations of the model reveals that; health education campaigns and SIT have significant impact on the spread of LF. It was established that, higher rate of mass treatment (MDA) significantly reduces the endemicity level in the community, but may delay the disease elimination/eradication as it seems to take much time. The disease elimination is only feasible if there is a combination of MDA, health education campaigns and incorporation of SIT in control program of LF. Moreover, the study suggests that, to keep the spreading of the disease under control, health education campaign should be increased through mass media (Televisions, radio, magazines as well as mobile networks like cell phones) from towns to villages. The model presented is not exhaustive; it can be extended to incorporate management of chronically affected individuals as well as to evaluate the cost effectiveness of these control strategies.

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