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OPTIMIZATION OF CASP-CUSUM SCHEMES BASED ON TRUNCATED ERLANGIAN DISTRIBUTION

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Abstract - Among several techniques to control the quality, acceptance sampling plans are introduced mainly to accept or reject the lots of finished products. Such type of techniques are involved in the product which are destructive where 100% inspection is not possible such as bullets, batteries, bulbs and so on. In this paper we consider CASP-CUSUM Schemes based on the assumption that the continuous variable under consideration follows a truncated Erlangian Distribution. The Erlangian Distribution plays a vital role in Statistical Quality Control, particularly in estimating reliability by considering its distribution. Optimum CASP-CUSUM Schemes are suggested based on numerical results obtained.
Key words: CASP-CUSUM Schemes, type C, OC Curve, ARL, Truncated Erlangian Distribution.

I. INTRODUCTION

In the competitive situation or phenomenon the reliability or quality of a product depends on several factors such as quality of materials, men, machines, management and equipments, the skill of the persons who involved in the process of production and the inspection of the final product. In past research the term “quality” defines in different dimensions, particularly with regard to consumer point of view, system designer’s point of view etc. Particularly in consumer’s point of view durability, safety, low-cost, the degree of satisfaction etc are the major characteristics which determine the quality of a product. Where as in producer’s point of view the degree of profit and the degree of low cost of production are the major properties that determine the quality. The term quality is closely associated with reliability of the product.

Among several techniques to control the quality, Acceptance Sampling Plans are introduced mainly to accept or reject the lots of finished products. Such types of techniques are popularly used where testing involves destruction, for instance, in the manufacturing of Crackers, Bullets, Batteries, Bulbs and so on, it is impossible to go for 100% inspection. Even though these techniques of Acceptance Sampling Plans do not have direct impact on controlling the quality of the product they have a lot of indirect effects to improve the same. For instance, a particular product is continuously rejected for lack of quality, automatically the producer strives to improve the quality of the product, otherwise, the consumer will opt for another better quality product.

Obviously, if the reliability of the product increases then the quality of the product will also increases, because in producer’s point of view, the probability that an item or a machine or a product will survive for a longer period of time. It states that a better quality of a product. It can be evidently observed that the quality of the product depends on the functioning of the system. The quality of the product is usually defer from time to time and also the quality defers from product to product in the same manufacturing process. For example in case of electric bulbs the durability of an electric bulb will changes from bulb to bulb. Thus quality is the random phenomena.

Lonnie. C. Vance, [6] considered Average Run Length of cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart. To determine the parameter of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are considered.

Hawkins, D.M. [3] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for specific parameter values and the out of control ARL's of location and scale CUSUM Charts.

Kakoty.S., Chakraborthy A.B. [5] proposed CASP – CUSUM charts under the assumption that the variable under study follows a Truncated Normal distribution. Generally truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or below designated tolerance limits. It is worthwhile to note that any continuous variable be first approximated as an exponential variable. Vardeman.S, Di-ou Ray [9] was introduced CUSUM control charts under the restriction that the values regard to quality are exponentially distributed. Further the phenomena under study is the occurrence of rate of rare events and the inter arrival times for a homogenous poison process are identically independently distributed exponential random variables.

YI Dai, Yunzhao Luo, Zhonghua, Li and Zhaojun Wang [10] recommended a more generalized multivariate CUSUM (MCUSUM) control charts which are usually called adaptive MCUSUM Control Charts which not only operate without any pre-knowledge about the process shift, but also achieve an overall approximately optimal performance at each point in a broader range of mean shifts. Finally it is concluded by addressing some relevant issues such as super imposing Shewart Control Limit on their AMCUSUM Chart will perform more efficiently for large mean shifts just like combine Shewart CUSUM Charts.

Muhammad Riaz, Nasir Abbas and Ronald J.M.M Does [7] proposed two Runs rules schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Akthar P.Md and Sarma K.L.A.P [1] proposed an optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluated L(O) L'(O) and Probability of Acceptance and also optimized CASP-CUSUM Schemes based on numerical results.

In the present paper it is proposed CASP-CUSUM Charts when the variable under study follows truncated Erlangian Distribution. Thus it is more worthwhile to study some interesting characteristics of this distribution.

A continuous random variable X assuming non-negative values is said to have Erlangian Distribution with parameters λ and M, its probability density function is given by:

$$f(x) = \frac{e^{-M\lambda x}(M\lambda)^M x^{M-1}}{(M-1)!} \qquad 0 < x < \infty$$

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$\lambda > 0$

$= 0$ Otherwise (1.1)

The random variable X is said to follow a truncated Erlangian distribution as:

$$f_B(x) = \frac{e^{-M\lambda x}(M\lambda)^M x^{M-1}}{(M-1)!\left[1-(M\lambda)^M e^{-M\lambda B}\left\{\sum_{j=1}^{M}\frac{B^{M-j}}{(M-j)\,!\,(M\lambda)^j}\right\}\right]} \quad (1.2)$$

$= 0$ otherwise

Where B is the truncation point of the Erlangian distribution.

The Erlangian distribution is useful when the production of cars where the production can be done in different stages/units like manufacturing of engine, body building, and manufacturing of tyres which are done independently in different units of production and ultimately assembled in the assembly unit, and tested in quality control unit and so on.

II. DESCRIPTION OF THE PLAN AND TYPE-C OC CURVE

The procedure in brief is given below

Beattie, [2] have suggested the method for constructing the continuous acceptance sampling plan. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum(X_i - k)$, X_i 's(i=1,2,3,…..) are distributed independently and k is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies in the of the return chart, the product is rejected, subject to the following assumptions.

I. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'.

II. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection. The procedure in brief is given below.

III. Start plotting the CUSUM at 0.

IV. The product is accepted when $S_m = \sum(X_i - k) < h$; when $S_m < 0$, return cumulative to 0.

V. When $h < S_m < h+h'$ the product is rejected: when S_m crosses h, i.e., when $S_m > h+h'$ and continue rejecting product until $S_m > h+h'$ return cumulative to $h+h'$.

The Type – C, OC function, which is defined as the probability of acceptance of an item as a function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P_A is given by

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$$P_A = \frac{L(0)}{L(0)+L'(0)} \tag{2.1}$$

Where L (0) = Average Run Length in acceptance zone and
$L'(0)$ = Average Run Length in rejection zone.

Page, E.S [8] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1-P(0)} \tag{2.2}$$

$$L'(0) = \frac{N'(0)}{1-P'(0)} \tag{2.3}$$

Where P(0) = Probability for the test starting from zero on the normal chart,
N(0) = ASN for the test starting from zero on the normal chart,

$P'(0)$ = Probability for the test on the return chart and
$N'(0)$ = ASN for the test on the return chart.

He further obtained integral equations for the quantities
P (0), N (0), $P'(0)$, $N'(0)$ as follows :

$$P(z) = F(k-z) + \int_0^h p(y) f(y+k-z)dy, \quad \ldots.. \tag{2.4}$$

$$N(z) = 1 + \int_0^h N(y) f(y+k-z)dy, \quad \ldots.. \tag{2.5}$$

$$P'(z) = \int_{k+z}^{B} f(y)dy + \int_0^h P'(y) f(-y+k+z)dy \ \ldots\ldots \tag{2.6}$$

$$N'(z) = 1 + \int_0^h N'(y) f(-y+k+z)dy, \ldots\ldots. \tag{2.7}$$

$$F(x) = 1 + \int_A^h f(x)dx :$$

$$F(k-z) = 1 + \int_A^{k-z} f(y)dy,$$

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and z is the distance of the starting of the test in the normal chart from zero.

III. METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(x) = Q(x) + \int_c^d R(x,t)\, F(t)\, dt \qquad (3.1)$$

Where $F(x) = P(z)$

$Q(x) = F(k - z)$

$R(x,t) = f(y + k - z)$

Let the integral $I = \int_c^d f(x)\, dx$ be transformed to

$$I = \frac{d-c}{2}\int_c^d f(y)\, dy = \frac{d-c}{2}\sum a_i\, f(t_i) \qquad (3.2)$$

Where $y = \frac{2x-(c-d)}{d-c}$ where a_i's and t_i's respectively the weight factor and abscissa for the Gauss-Chibyshev polynomial, given in Jain M.K. and et al [4] using (3.1) and (3.2), (2.4) can be written as

$$F(x) = Q(x) \quad \frac{d-c}{2}\sum a_i\, R(x,t_i) F(t_i) \qquad (3.3)$$

Since equation (3.3) should be valid for all values of x in the interval (c,d), it must be true for $x = t_i$. i = 0 (1) n then obtain

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i\, R(t_j,t_i) F(t_i) \qquad (3.4)$$

$J = 0\ (1)\ n$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i$, i – 0 (1) n, in (2.4.4), we get

$$F_o = Q_o + \frac{d-c}{2}\left[a_o R(t_o,t_o)F_o + a_1 R(t_0,t_1)F_1 + \ldots a_n R(t_0,t_n)F_n)\right]$$

$$F_1 = Q_1 + \frac{d-c}{2}\left[a_o R(t_1,t_o)F_o + a_1 R(t_1,t_1)F_1 + \ldots a_n R(t_1,t_n)F_n)\right]$$

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$$F_n = Q_n + \frac{d-c}{2}\left[a_o R(t_n,t_o)F_o + a_1 R(t_n,t_1)F_1 + \ldots a_n R(t_n,t_n)F_n)\right] \qquad (3.5)$$

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In the system of equations except F_i, i = 0, 1,…………… n are known and hence can be solved for F_i. We solve the system of equations by the method of Iteration. For this we write the system (3.5) as

$$[1-Ta_0R(t_o,t_0)]F_0 = Q_0 + T[a_oR(t_0,t_o)F_o + a_1R(t_0,t_1)F_1 + ...a_nR(t_0,t_n)F_n)]$$

$$[1-Ta_1R(t_1,t_1)]F_1 = Q_1 + T[a_0R(t_1,t_o)F_o + a_1R(t_1,t_1)F_1 + ...a_nR(t_1,t_n)F_n)]$$

…… …… …… ……
…… …… …… ……

$$[1-Ta_nR(t_n,t_n)]F_n = Q_n + T[a_oR(t_n,t_o)F_o + a_1R(t_n,t_1)F_1 + ...a_nR(t_n,t_n)F_n)] \quad \ldots (3.6)$$

Where $T = \frac{d-c}{2}$.

To start the Iteration process, let us put $F1 = F2 = \ldots. Fn = 0$ in the first equation of (3.6), we then obtain a rough value of Fo. Putting this value of Fo and $F2 = F_3 \ldots. Fn = 0$ in the second equation, we get a rough value F_1 and so on. This gives the first set of values F_i, I = 0,1,2,………., n which are just the refined values of $Fi, I = 0,1,2, \ldots.., n$. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions $P'(0), N(0), N'(0)$ can be obtained. We developed computer program to solve these equations and get the following tables

IV. COMPUTATION OF ARL's P (A)

TABLE – 4.1

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.4 h= 0.25 h'=0 .25

B	L(0)	L'(0)	P(A)
2.4	1.40117	1.11115	0.5577211
2.3	1.46393	1.12734	0.5649469
2.2	1.54436	1.14757	0.5737005
2.1	1.65044	1.17334	0.5844790
2.0	1.79566	1.20699	0.5980259
1.9	2.00473	1.25226	0.6155170
1.8	2.32827	1.31558	0.6389596
1.7	2.88853	1.40887	0.6721576
1.6	4.07485	1.55662	0.7235596
1.5	**8.14593**	**1.61970**	**0.8174028**

TABLE - 4.2

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.4 h= 0.5 h'= 0.5

B	L(0)	L'(0)	P(A)
2.8	1.62360	1.16675	0.5818620
2.7	1.71609	1.18814	0.5908943
2.6	1.83449	1.21428	0.6017155
2.5	1.99075	1.24678	0.6148982
2.4	2.20532	1.28798	0.6312991
2.3	2.51647	1.34152	0.6522749
2.2	3.00487	1.41324	0.6801262
2.1	3.87470	1.51321	0.7191476
2.0	5.83539	1.66023	0.7785067
1.9	**14.16858**	**1.89364**	**0.8821057**

TABLE - 4.3

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.4 h= 0.75 h'=0.75

B	L(0)	L'(0)	P(A)
3.1	1.80977	1.20849	0.5996075
3.0	1.92667	1.23299	0.6097715
2.9	2.07639	1.26253	0.6218746
2.8	2.27432	1.29870	0.6365266
2.7	2.54707	1.34374	0.6546379
2.6	2.94505	1.40103	0.6776337
2.5	3.57673	1.47584	0.7079039
2.4	4.72549	1.57679	0.7498066
2.3	7.43977	1.71907	0.8123049
2.2	**21.29356**	**1.93175**	**0.9168258**

TABLE - 4.4

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.4 h= 1 h'= 1

B	L(0)	L'(0)	P(A)
3.3	2.04441	1.25585	0.6194687
3.2	2.20135	1.28495	0.6314278
3.1	2.40571	1.31993	0.6457172
3.0	2.68193	1.36259	0.6631024
2.9	3.07452	1.41550	0.7125000
2.8	3.67410	1.48253	0.7495518

2.7	4.69766	1.56963	0.8018848
2.6	6.82641	1.68655	0.8821630
2.5	**13.85116**	**1.85020**	**0.9980039**

TABLE - 4.5

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.6 h= 0.25 h'=0.25

B	L(0)	L'(0)	P(A)
2.4	1.42415	1.11115	0.5617292
2.3	1.49069	1.12734	0.5693941
2.2	1.57603	1.14757	0.5786574
2.1	1.68867	1.17334	0.5900286
2.0	1.84299	1.20699	0.6042637
1.9	2.06539	1.25226	0.6225463
1.8	2.40995	1.31558	0.6468745
1.7	3.00742	1.40887	0.6809837
1.6	4.27484	1.55682	0.7330395
1.5	**8.63863**	**1.61970**	**0.8260050**

TABLE - 4.6

VALUES OF ARLs AND TYPE –C OC CURVES when

M= 3 λ = 0 .1 k = 0.6 h= 0.5 h'=0.5

B	L(0)	L'(0)	P(A)
2.8	1.64174	1.16675	0.5845631
2.7	1.73705	1.18814	0.5938261
2.6	1.85910	1.21428	0.6049042
2.5	2.02021	1.24678	0.6183706
2.4	2.24150	1.28798	0.6350794
2.3	2.56253	1.34152	0.6563771
2.2	3.06662	1.41324	0.6845350
2.1	3.96479	1.51321	0.7237666
2.0	5.99060	1.66023	0.7830001
1.9	**14.61233**	**1.89364**	**0.8852752**

VI. CONCLUSION

At the hypothetical values of the parameters λ, M k, h and h' given at the top of each table, we determine optimum truncated point B at which P(A) the probability of accepting an item is maximum and also obtained ARL's values which represents the acceptance zone L(O) and rejection zone L'(O) values. The values of truncated point B of random variable X, L(O) L'(O) and the values for Type – C OC Curve, i.e. P(A) are given in columns I, II, III and IV respectively.

From the above tables 4.1 to 4.6 we made the following conclusions

(i) From table 4.1 to 4.4, it was observed that the values of L(O), L'(O) and P(A) were increased as the value of truncated point decreased thus the truncated point of the random variable and the various parameters for CASP-CUSUM are inversely related.

(ii) And also we observed that it can be minimize the truncated point B by increasing value of k

(iii) From table 4.1 to 4.4, it is observed that the truncated point B changes from 2 to 1.9 and P (A) is as $h \rightarrow 1$ maximum i.e. 0.999993. Thus truncated point B and h are inversely related but h and P (A) are positively related.

(iv) From table 4.5 to 4.8 it can be observed that the truncated point B of the random variable X decreases from 5.0 to 2.7 as $h \rightarrow 1$, while the values of L(O) decreases from 13670. 79688 to 3186.33838 and rejection zone values changes from 1.2110069 to 0.0124716 where as the probability of acceptance P (A) changes from 0.9999114 to 0.999961 thus the hypothetical value h and truncated point B inversely related, while the values L(O) , L'(O) and P(A) are positively related.

(v) From tables 4.1 to 4.12 it can be observed that at the maximum level of probability of acceptance P(A) the truncated point B changes from 2 to 1.9 thus inverse relationship can be identified between B and P(A).

(vi) The observations mentioned in the above (i) to (v) with regards to the nature of ARL's with respect to h and k are in agreement with practical analogy.

(vii) The various relations exhibited among the ARL's and Type –C OC Curves with the parameters of the CASP-CUSUM based on the above table 4.1 to 4.12 are observed from the following table No. 5.1

Table No.5.1

B	λ	M	k	h	h'	L (O)	L'(O)	P(A)
1.5	0.1	3	0.4	0.25	0.25	8.14593	1.61970	0.8174028
2.0	0.1	3	0.4	0.5	0.5	14.16858	1.89364	0.8821057
2.2	0.1	3	0.4	0.75	0.75	21.29356	1.93175	0.9168258
2.5	**0.1**	**3**	**0.4**	**1**	**1**	**13.85116**	**1.85020**	**0.9980039**
1.5	0.1	3	0.6	0.25	0.25	8.63863	1.61970	0.8260050
1.9	0.1	3	0.6	0.5	0.5	14.61233	1.89364	0.8852752

By observing the above table the value of P(A) reach their maximum i.e. **0.9980039** respectively, and the parameters of the distribution are

$$\begin{bmatrix} B = 2.5 \\ \lambda = 0.1 \\ M = 3 \\ k = 0.4 \\ h = 1 \\ h' = 1 \end{bmatrix}$$

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