Scientific Journal Impact Factor (SJIF): 1.711

e-ISSN: 2349-9745
p-ISSN: 2393-8161

International Journal of Modern Trends in Engineering and Research

www.ijmter.com

TRANSDUCER MASS LOADING EFFECT ON IDENTIFICATION OF JOINT STIFFNESS PARAMETER

Dr.Sanjay B.Ingole[1], Kanchan S.Ingole[2]

[1]Department of Mechanical Engg.,Government Polytechnic, Sakoli

[2]Department of Electronics Engg., Government Polytechnic, Nagpur

Abstract— The quality of measured frequency response functions (FRFs) is adversely affected by many factors, most significant sources being noise and systematic errors. It is also known that the accuracy and the reliability of various analyses using the measured FRFs depend strongly on the quality of measured data. This paper aims to remove one of the major systematic errors in measured FRFs, namely the mass loading effects of transducers. Sherman–Morrison identity has been used for the elimination of mass loading effects of transducers from measured FRFs. The eigenvalues are estimated from corrected FRF's.

The parameters describing the dynamic behavior of supports, bearings and joints are usually not well known. The dynamic model of the system may be affected much more by the usual simplifications of bearings and joints than by slight errors in geometry, mass and stiffness distribution in its main components. This leads to the correction of mathematical model of real systems via identification of local parameters which in this paper limited to joint stiffness parameter. The joint parameters are estimated from corrected eigenvalues; a well known method suggested by U.Pabst and P.Hagedorn has been used for parameter estimation.

Keywords- transducer, joint stiffness, frequency response function, natural frequency, accelerometer mass

I. INTRODUCTION

The geometry and the material properties of the main components of large systems can often be specified with high accuracy. Therefore the corresponding parts of the model do not need to be corrected. On the other hand, the parameters describing the dynamic behavior of supports, bearings and joints are usually not well known. The dynamic model of the system may be affected much more by the usual simplifications of bearings and joints than by slight errors in geometry, mass and stiffness distribution in its main components. This leads to the correction of mathematical model of real systems via identification of local parameters. Dynamic analysis of mechanical systems often leads to the eigenvalue problem. Eigen frequencies of beams and plates are sensitive to the location of attached mass. In the studies [4,5] the sensitivity of the eigen frequency of the elastic beam with respect to small changes in the location of the in-span support is considered.

Accurate dynamic mathematical model of a structure is essential for simulating reliably the dynamic characteristics. It is essential to remove the undesirable effects of transducer mass loading from measured FRFs. The adverse effect due to this is significant especially for light weight structures and it may be necessary to eliminate the side effects before the data are used for further analysis [11]. Cancellation of transducer mass loading effect can also be considered as a structural modification problem. The desired properties of the original system are obtained by removing accelerometer mass from the system (modifying the system with negative mass).Structural assemblies have to be joined in some way, by bolting, welding and riveting or by more complicated fastenings such as smart joints. It is known that the added common approaches used in identification of the joint properties. The first approach employs non-parametric identification methods and is widely used since no assumption about the properties of the joint is required. The second approach in modeling joints employs parametric models. In contrast with non-parametric models, in using the parametric models it is necessary to have a good understanding about the involved physics of a joint. W.L.Li [10] proposed a reduced order characteristic polynomial (ROCP) which can be used for updating or identifying joint stiffness parameter.

This paper deals with removing the effect of accelerometer mass from measured FRFs, aimed at improving the quality of the measured data. The validity of this approach is demonstrated using experimental data. The natural frequencies are estimated from corrected FRFs. and utilized in the method suggested by U.Pabst and P.Hagedorn [2] for estimation of joint stiffness parameter

II. CANCELLATION OF TRANSDUCER MASS LOADING

A problem with the measured FRF's is the mass loading effects of transducer which is mounted on the test structure. Transducer mass causes the natural frequency of the structure to shift from their correct values, hence introducing a systematic error in the measured FRFs. Generally, this effect is ignored in the analytical and experimental modeling process, based on the usual assumption that the transducer mass is negligible compared to that of the structure under test. However, when light weighted structures are investigated, this effect can be significant and it can be necessary to eliminate these undesirable side effects before the data are used for further analysis.

The method proposed in this paper is based on the Sherman-Morrison identity [11]] which allows a direct inversion of a modified matrix efficiently using the data related to the initial matrix and to the modifications. The objective of the paper is to remove the transducer mass loading effect from the measured FRFs. This is achieved by considering the original structure with the transducer mass as the modified structure. A negative mass is added to the structure as a modification so as to estimate the correct FRFs without the transducer mass loading effect.

If $[Q]^{-1}$ is the inverse of a non-singular square matrix [Q] and the modification is expressed as a product of two vectors such as $\{a\}\{b\}^T$, so that the modified matrix is given by –

$$[Q^*] = [Q] + \{a\}\{b\}^T$$

Then, the inverse $[Q^*]^{-1}$ can be calculated by using Sherman-Morrison formula as -

$$[Q^*]^{-1} = [Q]^{-1} – ([Q]^{-1}\{a\})(\{b\}^T[Q]) \;/\; (1+\{b\}^T[Q]^{-1}\{a\})$$

A general equation for dynamic stiffness matrix $[Z]=[K]-\omega^2[M]+j\omega[C]$ where [K],[M],[C] are stiffness, mass(including transducer mass) and damping matrices. Let ΔM be the transducer mass to be removed from the dynamic stiffness matrix. The modified system $[Z^*]=[Z]+[\Delta Z]$. The receptance matrix $[\alpha] = [Z]^{-1}$ and the modification matrix $[\Delta Z] =\{a\}\{b\}^T$, the FRF of the modified structure can be computed using the Sherman – Morrison as

$$[\alpha^*] = [Z^*]^{-1}=[\alpha]- ([\alpha]\{a\})(\{b\}^T[\alpha]) \;/\; 1+\{b\}^T[\alpha]\{a\}$$

$[\alpha^*]$ matrix contains desired FRF without the effect of transducer mass. If only one transducer mass modification m is considered at coordinate n alone, then the modification matrix can be expressed as,

$$[\Delta Z] = -\omega^2(-[\Delta M])= \{a\}.\{b\}^T$$

A practical way of removing the transducer mass loading effects can be obtained by using the formulation above at active coordinates only, i.e. excitation, response and modification co-ordinates. Now the general expression for the desired FRF α^*_{lm} can be expressed as [11],

$$\alpha^*_{lm} = \alpha^n_{lm} + \omega^2 m(\alpha^n_{nn}\, \alpha^n_{lm} - \alpha^n_{ln}\, \alpha^n_{nm}) \;/\; (1+ \omega^2 m\alpha^n_{nn}) \qquad (1)$$

where, l,m,n represent response, excitation and modification coordinate.

Above expression can be expressed in terms of accelerance A_{lm} $(A= - \omega^2 \alpha)$

$$A^{*}_{lm} = A^{n}_{lm} - m(A^{n}_{nn} A^{n}_{lm} - A^{n}_{ln} A^{n}_{nm})/(1-m A^{n}_{nn}) \quad (2)$$

For the driving point FRF, l=m=n, Eq.(2) then becomes,

$$A^{*}_{mm} = A^{m}_{mm} / 1-mA^{m}_{mm}$$

Above general expression Eq.(1) and Eq.(2) are valid irrespective of the modification coordinate being identical to the excitation or response coordinate.

III. IDENTIFICATION OF UNKNOWN BOUNDARY PARAMETERS (JOINT STIFFNESS)

There is a variety of different methods for identification of unknown system parameters. A method proposed by U.Pabst and P.Hagedorn has been used for the determination of the boundary parameter K_1 (torsional stiffness) and K_2 (translational stiffness). The method estimates the joint stiffness parameter from natural frequencies of the beam. The error in the measurement of natural frequencies produces error in the estimation of joint stiffness parameters. Considering the fact that the transducer mass causes the natural frequency of the structure to shift from their correct values, cancellation of transducer mass loading effect is inevitable to minimize the influence of systematic error in the measured FRFs and natural frequency. The method here will use modified natural frequencies obtained through cancellation of transducer mass loading effect for estimation of joint stiffness parameter.

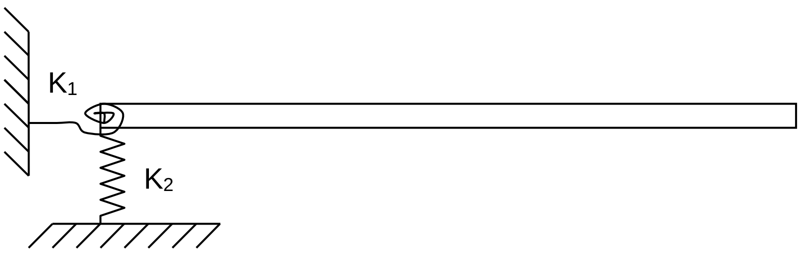

Figure 1. A cantilever beam elastically constrained at one end

Consider a cantilever beam elastically constrained at one end, the eigen value problem of this beam modeled as an Euler-Bernoulli beam is shown in Figure 1, for a beam with constant cross section the eigen value problem leads to the following characteristic equation proposed by U.Pabst and P.Hagedorn [2].

$$X_1 - (K_1/EI)X_2 + (K_2/EI)X_3 + (K_1/EI)(K_2/EI) X_4 = 0 \quad (3)$$

Where, $X_1 = \gamma^4(1-\cos\gamma L.\cosh\gamma L)$ $\quad X_2 = \gamma^3(\cos\gamma L.\sinh\gamma L + \sin\gamma L.\cosh\gamma L)$

$X_3 = \gamma(\cos\gamma L.\sinh\gamma L - \sin\gamma L.\cosh\gamma L)$ $\quad X_4 = 1+\cos\gamma L.\cosh\gamma L$

$\gamma^2 = (2\pi f)\sqrt{\frac{\rho A}{EI}}$, EI is bending stiffness, L is the length of beam, ρA is mass distribution and f is modified natural frequency (neglecting transducer mass loading).

For each experimentally and or analytically determined natural frequency f_i (i = 1 to N), Eq.(3) leads to a non-linear equation for the two unknown parameters K_1 and K_2. Instead of dealing with this over determined system of equations, we can also determine the parameters by taking pairs f_i, f_j, $i \neq j$ of natural frequencies and computing ${}_iX_1$ to ${}_iX_4$ and ${}_jX_1$ to ${}_jX_4$. Substituting these into Eq.(3) and rearranging leads to –

$$K_1/EI = \{ {}_iX_1 + (K_2/EI)\, {}_iX_3 \} / \{ {}_iX_2 - (K_2/EI)\, {}_iX_4 \} \quad (4)$$

$$K_2/EI = \left(P_{ij} \pm \sqrt{\left(P_{ij}^{\,2} + 4Q_{ij}\right)} / 2 \right) \quad (5)$$

With, $P_{ij}= ({}_jX_2\, {}_iX_3 - {}_iX_2\, {}_jX_3 + {}_jX_1\, {}_iX_4 - {}_jX_4\, {}_iX_1)/({}_jX_4\, {}_iX_3 - {}_jX_3\, {}_iX_4)$ and

$Q_{ij} = ({}_jX_2\, {}_iX_1 - {}_jX_1\, {}_iX_2)/({}_jX_4\, {}_iX_3 - {}_jX_3\, {}_iX_4)$

The negative roots in Eq.(5) is associated with a negative and, therefore, useless value of the stiffness K_2.

IV. EXPERIMENTAL STUDY

Consider a cantilever beam elastically constrained at one end as shown in Figure 1, the beam is made of steel (E=2.07x1011 N/m2).Experimental modal analysis carried out for a steel cantilever beam with constant rectangular cross section with length L=990 mm, width b=40 mm and height h=12 mm. FRFs are measured with an accelerometer having total mass of 25 gm. (including magnetic base) (Figure 2).The first five measured natural frequencies were f_1 = 9.748 Hz; f_2 = 61.316 Hz, f_3 = 172.23 Hz, f_4 = 338.63 Hz, f_5 = 561.93 Hz.

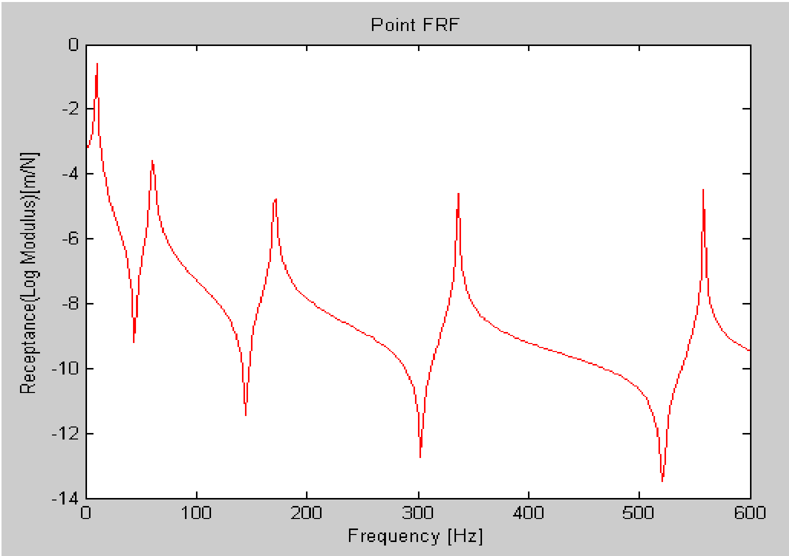

Figure 2. FRF measured with an accelerometer mass of 25 gm

From the measured FRFs, desired FRFs are generated (Figure 3) by cancellation of transducer mass loading effect from Sherman-Morrison identity (Eq.1) . The objective here is to determine correct natural frequencies (by filtering out undesirable effect of transducer mass loading) which are then used to estimate joint stiffness parameter. The proposed method is applied to remove transducer mass loading effect from measured FRFs, this is achieved by considering the transducer mass as a modification to the original structure and idea here is to modify the structure again, but this time the modification is to remove the mass from the structure i.e. a negative transducer mass is added to the structure as a modification in order to estimate correct FRFs without the mass loading effect. Figure 3 and Figure 4 shows point and transfer FRF α_{11} and α_{23} respectively showing measured FRF (with accelerometer mass of 25 gm); corrected FRF is compared to their exact and the measured counterparts. The corrected FRF match perfectly with the target values, as expected the resonance frequency of the system measured with accelerometer mass is lower than that without the accelerometer mass.

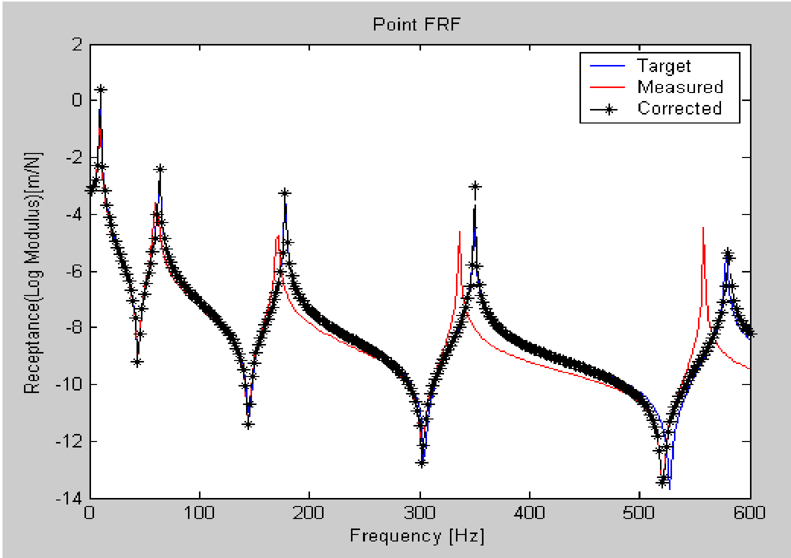

Figure 3. Comparison of measured (with accelerometer mass of 25 gm), target (without mass) and corrected FRF α_{11}

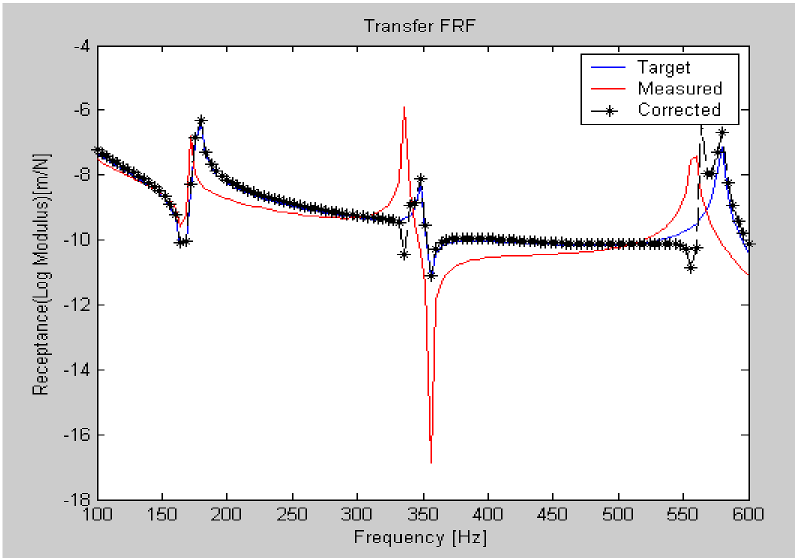

Figure 4. Comparison of measured (with accelerometer mass of 25 gm), target(without mass) and corrected Transfer FRF α23

The first five estimated natural frequencies by eliminating transducer mass loading effect were f_1 = 10.225 Hz; f_2 = 64.081 Hz, f_3 = 179.47 Hz, f_4 = 351.93 Hz, f_5 = 582.68 Hz.

4.1. Identification of Joint Stiffness Parameter

The set of natural frequencies measured and estimated with and without transducer mass loading effect are given in Table 1.

Table 1. Measured and estimated natural frequencies

S.No.	Natural frequencies measured with accelerometer mass of 25 gm. (Hz)	Natural frequencies estimated without accelerometer mass. (Hz)	Error (%)
f_1	9.748	10.225	4.89
f_2	61.316	64.081	4.51

f_3	172.23	179.47	4.20
f_4	338.63	351.93	3.92
f_5	561.93	582.68	3.69

Now for the identification of joint stiffness parameter K_1 and K_2 via Eq. (4) and Eq.(5), above set of natural frequencies are used. As the equation needs pairs of frequencies; four pairs of frequencies f_2, f_3; f_2, f_4; f_2, f_5; f_3, f_4; are taken to estimate K_1 and K_2. The stiffness parameter K_1 and K_2 identified with each of these pairs are shown in Table 2.

Table 2. Identified joint stiffness parameters

Natural frequencies	With accelerometer mass		Without accelerometer mass	
	Torsional stiffness K_1 (10^4 Nm/rad)	Translational stiffness K_2 (10^7 N/m)	Torsional stiffness K_1 (10^6 Nm/rad)	Translational stiffness K_2 (10^7 N/m)
f_2, f_3	4.900	8.363	4.157	8.497
f_2, f_4	4.892	9.260	4.171	8.473
f_2, f_5	4.895	9.663	4.198	8.426
f_3, f_4	5.018	12.394	4.220	8.457
Mean Value	4.926	9.920	4.186	8.463

From above table it can be seen that the torsional stiffness is more sensitive to small change in natural frequencies. Sensitivity analysis has not been discussed in this paper. It is very essential to have correct measurement of natural frequencies in the proposed method.

V. CONCLUSIONS

A method based on Sherman-Morrison formula is presented in this paper to eliminate the transducer mass loading effects from measured FRFs. The natural frequencies obtained with and without accelerometer mass are used in the proposed equations to estimate joint stiffness parameters. The effect of transducer mass on measured receptance was modelled; transducer inertia caused shift in natural frequencies in the range 3.7% to 4.9% (for first five natural frequencies) which significantly influenced the boundary parameters.

Typical results presented in Figure 3 and Figure 4 show expected trend in the sense that the natural frequency of the system without the effect of the transducer mass shift to higher frequency compared to the values measured in practice using accelerometer. The discrepancies between the measured and target FRFs may well be due to the rotational effects of the transducer inertia and the degree of this effect can be mode dependent.

REFERENCES

[1] D.J.Ewins : Modal Testing: Theory, Practice and Application. Research Studies Press; second edition, 2000.
[2] U.Pabst and P.Hagedorn, "Identification of Boundary Condition as a Part of Model Correction", J. of Sound and Vibration ,182(4), 565-575, 1995.
[3] G.M.L.Gladwell and H.Ahmadian, "Generic Element Matrices Suitable for Finite Element Model Updating"; J. of Mechanical System and Signal Processing, 9(6), 601-614, 1995.
[4] A.Gurgoze, "On the Sensitivities of the Eigenvalues of a Viscously Damped Cantilever Carrying a Tip Mass", J. of Sound and Vibration, 216(2), 215-225, 1998.
[5] J.M.M.Silve, "Cancellation of Mass-Loading Effects of Transducers and Evaluation of Unmeasured Frequency Response Functions", J. of Sound and Vibration, 236(5), 761-779, 2000.
[6] A.Sestieri, "Structural Dynamic Modification", Sadhana, Vol.25, part 3, 247-259, June 2000.
[7] T.K.Kundra, "Structural Dynamic Modifications via Models", Sadhana, Vol.25, part 3, 261- 276, June 2000.

[8] K.G.McConnell, "Transducer Inertia and Stinger Stiffness Effects an FRF Measurements", J. of MSSP ,14(4), 625-636, 2000.
[9] O.Cakar and K.Y.Sanliturk, "Elimination of Noise and Transducer Effects From Measured Response Data", Proceedings of ESDA,Istanbul Turkey, ESDA2002/APM-055,1-10, 2002.
[10] W.L.Li, "A New Method for Structural Model Updating and Joint Stiffness Identification", J. of MSSP, 16(1), 155-167, 2002.
[11] O.Cakar and K.Y.Sanliturk, "Elimination of Transducer Mass Loading Effects from Frequency Response Functions", J.of MSSP 19, 87-104, 2005.
[12] H.Ahmadian and Hassan Jalali, "Identification of Bolted Lap Joint Parameters in Assembled Structures", J. of MSSP, 2005.

