

## Eigenvalue Analysis of Subsynchronous Resonance Study in Series Compensated Wind Farm

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**Abstract**— This paper presents an eigenvalue analysis for detecting the potential of subsynchronous resonance (SSR) in single-cage induction generator based wind farm. Fixed speed single-cage induction generator based wind farms are considered for study of SSR. The wind farm is connected to the modified IEEE first bench mark system. Detailed model of the study systems are developed and eigenvalue analysis is carried out. It is found that the single-cage induction generator based wind farms are susceptible to SSR when they are connected to IEEE first bench mark system. In this case SSR is occurs at realistic levels of series compensation employed for single-cage induction generator based wind farm. The eigenvalue analysis results are validated using PSCAD/EMTDC software. Hence, eigenvalue analysis can be successfully utilized for investigation of SSR in wind farm connected to series compensated transmission line.

**Keywords**- Eigenvalue analysis, subsynchronous resonance (SSR), series compensation, PSCAD/EMTDC.

### I. NOMENCLATURE

$\delta_{tg}$	Tortional angle between the wind turbine and generator.
$\omega_t, \omega_g$	Angular speed of the wind turbine and the generator rotor respectively.
$H_t, H_g$	Inertia constant of the wind turbine and the generator rotor respectively.
$K_{tg}$	Shaft stiffness between wind turbine and the generator.
$D_{tg}$	Damping coefficient between wind turbine and the generator.
$T_w, T_g$	Mechanical torque input to turbine and Electromagnetic torque output of generator respectively.
$R_s, R_r$	Stator winding resistance and rotor cage resistance respectively.
$L_{s\sigma}, L_{r\sigma}$	Stator winding unsaturated leakage inductance and rotor cage unsaturated leakage inductance respectively.
$L_m$	Unsaturated magnetizing inductance.
$\omega_s$	Synchronous frequency (rad/sec).
$I_{ds}, I_{qs}$	$d$ - $q$ axis stator current of induction generator .
$V_{ds}, V_{qs}$	$d$ - $q$ axis voltage at induction generator terminals.
$I_{dr}, I_{qr}$	$d$ - $q$ axis rotor current of induction generator .
$X_{s\sigma}, X_{r\sigma}$	Stator and rotor leakage reactance respectively.
$X_m$	Mutual reactance between stator and rotor winding.
$I_d, I_q$	$d$ - $q$ axis current of series compensated transmission line .
$R$	Transmission line resistance
$L$	Transmission line inductance including transformer and infinite grid inductance.
$C$	Series capacitance
$V_{cd}, V_{cq}$	$d$ - $q$ axis voltage across series capacitor.
$V_{bd}, V_{bq}$	$d$ - $q$ axis voltage at infinite bus.
$K$	Series compensation level (%)

## II. INTRODUCTION

The most important change that takes place in 2012 was acceleration in the geographical shift of renewable energy investment. Up to 2013 the global wind installation rose 16% to almost 48.4 GW [1]. With this rapid growth of installed capacity of wind farms, the large wind turbine generators are integrated into electric power grids. The penetration of these wind energy into power system can be done through better utilization of existing infrastructure [2]. Now it is well known fact that, series compensation is an effective means of increasing transfer capability of an existing transmission [3-4].

The series capacitor compensation can produce a significant adverse effect called sub-synchronous resonance (SSR) in electrical networks in which electrical energy is exchanged with the generator shaft system in a growing manner, which may result in damage of the turbine-generator shaft system [4]. In 2009 Electric Reliability Council of Texas (ERCOT) reported SSR in wind farm. A severe damage was caused due to this event [5].

SSR is an electric power system condition where the electric network exchanges energy with turbine generator at one or more of the natural frequencies of the combined system below the synchronous frequency of the system. There are two aspects of SSR problem. i) *Self Excitation* involving, only rotor electric dynamics termed as induction generator effect (IGE), which may occur in all kind of power generating plants and other type of self excitation involves both rotor electrical and mechanical dynamics which is called torsional interaction (TI). TI effect is much more significant compared to the IGE [6]. This can cause shaft damage as experienced at Mohave generating station. ii) *Transient Torque or Torque Amplification (TA)* Torque Amplification (TA) is the phenomenon that results from system disturbances [7].

SSR in conventional power plants are well documented, but not much information is available for SSR in wind farm. Wind farm interaction with series compensated transmission line was discussed for the first time in [8]. Electrical power system with wind farm and series compensated transmission line is also subjected to subsynchronous resonance phenomenon [9]. However, for wind farm some important differences exist with respect to conventional turbine-generators like small to medium capacity plants, more (Even more than 100) turbine-generator units; generator ratings are few kw to 2-3 MW range; generator operates at lesser Speed; variable speed wind turbines (Due to variable wind speed); less generator output voltage; collector cables are required from wind turbine to collector points. (This length is in km and varied according to location of wind turbine) and different types of generator such as Synchronous or Asynchronous (Induction Generators either squirrel cage, wound rotor or doubly fed and Permanent Magnet etc.).

Most recent trend is to use full convertor based wind turbine generators. However, many large wind farms across USA, India and Australia are in operation or under construction which utilize induction generator based wind turbines [10]. Simplicity, reliability, robustness and cheapness are the major advantages of these wind turbines [11].

This paper presents eigenvalue analysis to detect the potential of SSR in induction generator based series compensated wind farm. The analysis is carried out considering single-cage induction generator and is performed with IEEE first bench mark system. The eigenvalue analysis results are validated using PSCAD/EMTDC software.

The paper has been organized as follows; section 3 describes the different SSR analytical tools. In section 4, complete system modeling is presented. Section 5 presents the eigenvalue analysis. PSCAD/EMTDC simulation is performed in Section 6 and conclusion is summarized in section 7.

## III. SSR ANALYTICAL TOOLS

There are several techniques available for the study of subsynchronous resonance in power systems. The most common techniques are:

1. Frequency scanning
2. Eigenvalue analysis

3. Electromagnetic transient simulation

**3.1. Frequency Scanning**

The frequency scanning technique is a fundamental technique for analysis of subsynchronous resonance. It involves the determination of the driving point impedance over the frequency range of interest as viewed from the neutral bus of the generator under study. In this technique, the equivalent resistance and reactance are computed by looking into the network from a point behind the stator winding of a particular generator, as a function of the frequency. Should there be a frequency at which the reactance is zero and resistance is negative, self-sustaining oscillations at that frequency would be expected due to the induction generator effect. Frequency scanning can sometimes provide information regarding the possible problems with torsional interaction and transient torques. This method is easy to use and is fast [6-7].

**3.2. Eigenvalue Analysis**

Eigenvalue analysis is extensively used for the study of torsional interaction and induction generator effect [6]. This analysis is studied through the linearized model of the power system. The procedure of the eigenvalue analysis includes:

1. Modeling of the power system network
2. Modeling of generator electrical circuits
3. Modeling of turbine-generator spring mass system
4. Calculation of eigenvalues of the interconnected systems

Real component of eigenvalues corresponding to the subsynchronous modes of the turbine-generator spring mass system indicates the severity of torsional interaction and real component of eigenvalues corresponding only to electrical system resonant frequencies reveals the severity of the induction generator effects problem.

The real part of the eigenvalue is a direct measure of the positive or negative damping for each mode. Eigenvalues are defined in terms of the system linear equations that are written in the following standard form.

$$\dot{x} = Ax + Bu \tag{1}$$

Then the eigenvalues are defined as the solution to the matrix equation

$$\det[\lambda U - A] = 0 \tag{2}$$

Where, the parameters  $\lambda$  are called the eigenvalues.

Eigenvalue analysis is attractive since it provides the frequencies and damping at each frequency for the entire system in a single calculation.

**3.3. Electromagnetic transient simulation**

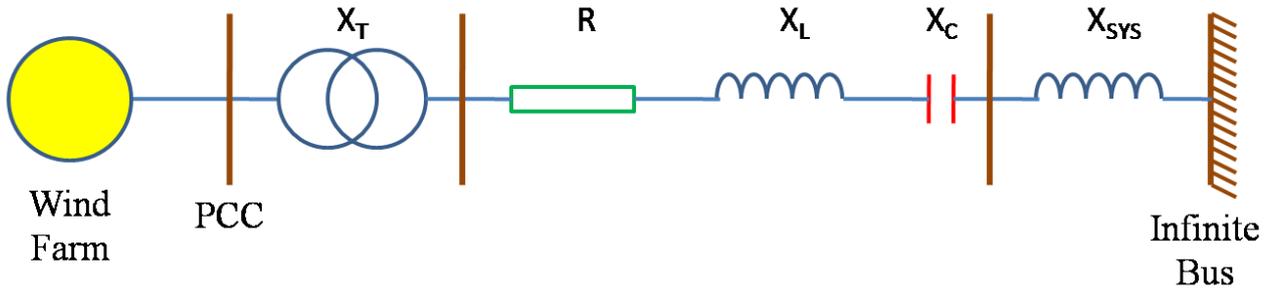
PSCAD / EMTDC software is found to be valid software for electromagnetic transient simulation. SSR analysis is carried out through a comprehensive eigenvalue analysis that is then reasonably validated through the electromagnetic transient simulation in PSCAD / EMTDC software by various literatures.

[12] Reported that, PSCAD/EMTDC proved to be a valuable tool in predicting the behavior of the wind turbine (WT), in selecting controller parameters and optimizing in general the control and operation of machine.

**IV. SYSTEM MODELING**

The IEEE first benchmark modified system for SSR studies [13] is as shown in Fig.1. The original benchmark system was proposed to facilitate the computer simulation of subsynchronous resonance in a power system that was based on the Navajo project 892.4 MVA synchronous generators connected to a 500 KV transmission system. In this paper, the conventional synchronous generator is

replaced with an equivalent capacity wind farm with induction generator. It is assumed that there are 'n' number of identical wind turbines and are connected to point of common coupling (PCC) through the collector cables.

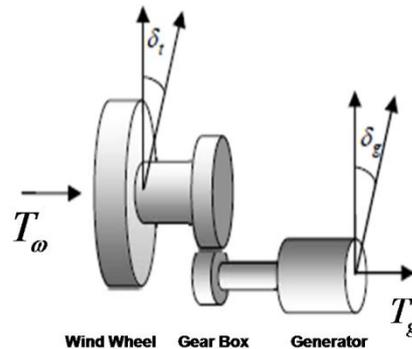


**Figure 1. IEEE First Bench Mark Modified System**

For modeling purpose, the system is divided into two sub-systems: i) wind farms and ii) transmission line. The wind farm is sub divided into two sub-systems: i) Mechanical drive train system and ii) Electrical generator system. The modeling of various subsystems is presented in following sections.

#### 4.1. Wind turbine drive train system model

In this paper a two mass drive train system is adopted, which is widely used for studying the power system stability including the wind turbine generator [14]. A Two mass drive train system is shown in figure 2.



**Figure 2. Two mass drive train system**

In two-mass model, one mass represents the summation of the mass of the blades and the hub whereas the other mass represents the electrical generator connected through the shaft and expressed in third order system, neglecting the self damping as follows[15]:

$$\frac{d}{dt}(\delta_{tg}) = \frac{d}{dt}(\delta_t) - \frac{d}{dt}(\delta_g) = \omega_t - \omega_g \quad (3)$$

$$2 H_t \frac{d}{dt}(\omega_t) = T_w - K_{tg} \delta_{tg} - D_{tg}(\omega_t - \omega_g) \quad (4)$$

$$2 H_g \frac{d}{dt}(\omega_g) = K_{tg} \delta_{tg} + D_{tg}(\omega_t - \omega_g) - T_g \quad (5)$$

The above equations are written in the linear state space form as:

$$\dot{x}_1 = A_1 x_1 + B_1 u_1 \quad (6)$$

Where,

$$x_1 = [\Delta\omega_t \quad \delta_{tg} \quad \Delta\omega_g]^T \quad \text{and} \quad u_1 = [T_w \quad T_g]^T$$

#### 4.2. Aggregated Model of Wind Turbine Drive Train System

In wind farm large numbers of wind turbines are connected to a point of common coupling (PCC). It is assumed that the disturbances on the system affect the performance of the system identically.

#### 4.3. Induction Generator Model

The equivalent circuit of single cage induction generator is shown in figure 3. The mathematical model of a single cage generator is developed in the synchronously rotating  $d$ - $q$  reference frame.

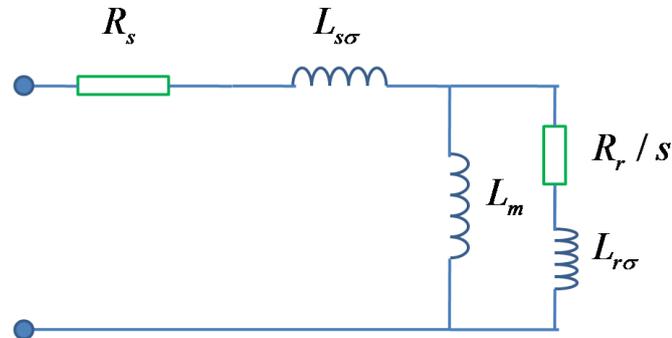


Figure 3. Single cage Induction Generator Equivalent Circuit

This model is based on the generalised flux linkage equation, which is widely used in the literature for the stability study of the wind turbine generator system. This model is expressed by the set of differential equations written in  $d$ - $q$  frame of reference as follows [16]:

$$\frac{1}{\omega_s} \frac{d}{dt} \lambda_{ds} = -R_s I_{ds} + \lambda_{qs} - V_{ds} \quad (7)$$

$$\frac{1}{\omega_s} \frac{d}{dt} \lambda_{qs} = -R_s I_{qs} - \lambda_{ds} - V_{qs} \quad (8)$$

$$\frac{1}{\omega_s} \frac{d}{dt} \lambda_{dr} = -R_s I_{dr} + s \lambda_{qr} \quad (9)$$

$$\frac{1}{\omega_s} \frac{d}{dt} \lambda_{qr} = -R_r I_{qr} - s \lambda_{dr} \quad (10)$$

Where,

$$\lambda_{ds} = X_s I_{ds} + X_m I_{dr}$$

$$\lambda_{qs} = X_s I_{qs} + X_m I_{qr}$$

$$\lambda_{dr} = X_r I_{dr} + X_m I_{ds}$$

$$\lambda_{qr} = X_r I_{qr} + X_m I_{qs}$$

$$X_s = X_{s\sigma} + X_m$$

$$X_r = X_{r\sigma} + X_m$$

The above equations are written in the linear state space form as:

$$\dot{x}_2 = A_2 x_2 + B_2 u_2 \quad (11)$$

Where,

$$x_2 = [I_{ds} \ I_{qs} \ I_{dr} \ I_{qr}]^T \quad \text{and} \quad u_2 = [\omega_g \ V_{ds} \ V_{qs}]^T$$

#### 4.4. Series Compensated Transmission Line Model

The series compensated line in this paper is adapted from IEEE first benchmark system [13]. The series compensated transmission line is shown in figure 4.

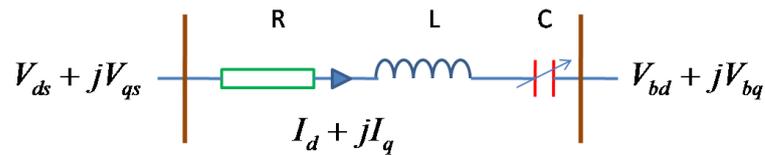


Figure 4. Series Compensated Transmission Line

The mathematical model of this line is developed in the synchronously rotating  $d$ - $q$  reference frame are as follows [17]:

$$L \frac{d}{dt} I_d = V_{ds} - R I_d + \omega_s L_q - V_{cd} - V_{bd} \quad (12)$$

$$L \frac{d}{dt} I_q = V_{qs} - R I_q + \omega_s L_d - V_{cq} - V_{bq} \quad (13)$$

$$C \frac{d}{dt} V_{cd} = I_d + \omega_s C V_{cq} \quad (14)$$

$$C \frac{d}{dt} V_{cq} = I_q - \omega_s C V_{cd} \quad (15)$$

Where,

$L$  = Transmission line inductance including transformer and infinite grid inductance.

The above equations are written in the linear state space form as:

$$\dot{x}_3 = A_3 x_3 + B_3 u_3 \quad (16)$$

Where,

$$x_3 = [V_{ds} \ V_{qs} \ I_d \ I_q \ V_{cd} \ V_{cq}]^T \quad \text{and}$$

$$u_3 = [I_{ds} \ I_{qs} \ V_{bd} \ V_{bq}]^T$$

## V. EIGENVALUE ANALYSIS

A wind farm connected to 500 kV transmission line is considered for eigenvalue analysis. The parameters of 2 MW single-cage induction generator based wind turbine and transmission system are given in Table 1 and Table 2 respectively.

Several identical wind turbines are represented by a single equivalent unit for analysis. Eigen values are calculated for 45%, 60% and 75% compensation levels. Table 3 depicts the eigenvalues for selected modes such as electrical mode, electro-mechanical mode, rotor mode, torsional mode (mode 1), and monotonic mode (mode 0).

Table 1. Data for a 2-MW Induction Generator Based Wind Turbine [16]

Rated Power	2.0 MW
Rated Voltage	690 V
Rated frequency, $f$	60 Hz
Nominal slip, $s$	0.015
Magnetic Reactance, $X_m$	3.8 p.u
Stator Resistance, $R_s$	0.048 p.u
Stator Reactance, $X_s$	0.075 p.u
Rotor Resistance, $R_r$	0.018 p.u
Rotor Reactance, $X_r$	0.12 p.u
Generator Rotor Inertia Constant, $H_g$	0.5 s
Turbine Inertia Constant, $H_t$	4.5s
Shaft Stiffness, $K_{tg}$	0.55 p.u

**Table 2. 500 KV Transmission System Parameters [13]**

Positive sequence Resistance	5.60286 $\Omega$
Positive sequence Resistance	140.0715 $\Omega$
Capacitive Reactance: k(X)	k being varied from (0.20 to 0.70)
Positive sequence system thevenin	16.808 $\Omega$

**Table 3. Eigenvalues for Various Modes and Degree of Compensation**

Different Modes	Degree of Series Compensation		
	45%	60%	75%
Electrical	0.5921±89.9i	0.6984±97.86i	0.7664±107.7i
Electro-mechanical	-3.875±31.985i	-4.014±38.98i	-4.986±54.665i
Rotor	-332.87±9.321i	-357.32±11.65i	395.62±15.65i
Mode 1	-0.4532±3.875i	-0.502±4.986i	-0.698±6.127i
Mode 0	-6.954	-7.023	-7.983

From the Table III, it is found that electrical mode is highly sensitive, which becomes unstable with increasing degree of series compensation.

For instance, with 45% compensation, the electrical mode eigenvalues are  $0.5921 \pm 89.9i$ , which represents an unstable operating condition. In case of power output with higher degree of series compensation, the electrical mode eigenvalues are  $0.6984 \pm 97.86i$  and  $0.7664 \pm 107.7i$  for 60% and 75% compensation respectively.

## VI. PSCAD/EMTDC SIMULATION

In order to investigate the possibility of pure electrical resonance, a single lumped mass representation was used for the turbine-generator system instead of the standard two-mass representation in this particular simulation. This is required for de-coupling the effect of any torsional modes during electrical self-excited resonance. Series capacitors corresponding to 60% compensation level are kept in service.

Figure 5 shows the electromagnetic torque and speed responses. Subsynchronous oscillations in the electromagnetic torque tend to reach up to 0.6 p.u. Currents and voltages are the principal interacting variables during electrical self-excitation and as a result, the generator electromagnetic dynamics is influenced. Electromagnetic torque and speed responses in Fig. 5 indicate that the wind turbine-generator can experience severe oscillations from which it never recovers.

Figure 6 shows the change in generator electromagnetic torque due to series capacitor insertion for varying compensation and aggregation levels. It can be observed that, higher series compensation level leads to a greater change in electromagnetic torque response. Secondly, as the number of machines aggregated increases, the amplitude of torque response at higher compensation levels increases drastically.

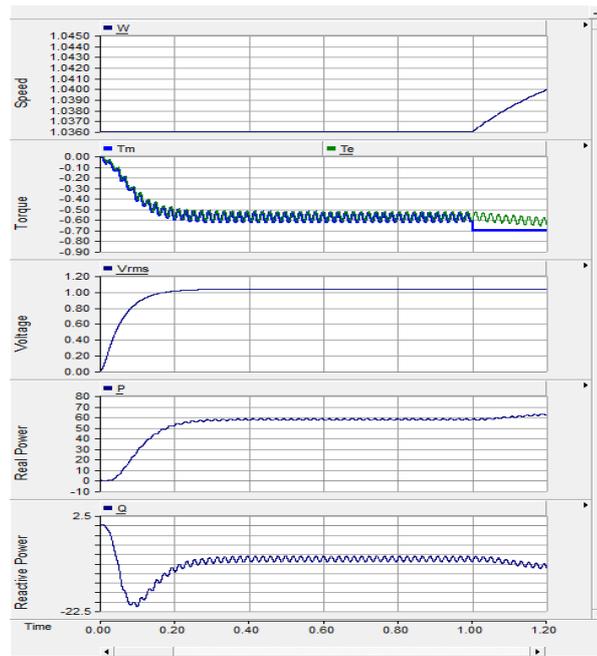


Figure 5. Mechanical speed and electromagnetic torque during starting with 60% series compensation in service, illustrating the possibility of electrical resonance.

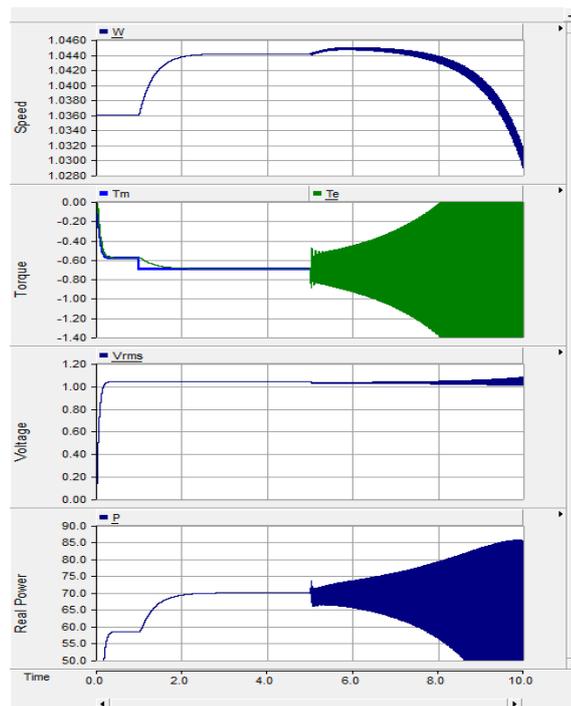


Figure 6. Higher Degree of Series compensation and aggregation level

## VII. CONCLUSION

In this paper, the potential for SSR occurrence is studied for single-cage induction generator based wind farms connected to series-compensated transmission line.

The detailed mathematical model of study system is developed and eigenvalue analysis is carried out for various compensation levels. Results obtained from eigenvalue analysis are validated through PSCAD/EMTDC software.

From the simulation results it is observed that the amplitude of electromagnetic torque response to transient events increases with the compensation level. Higher compensation levels also lead to greater possibility of SSR.

It is found that the induction generator based wind farms connected to IEEE First SSR Benchmark system are susceptible to SSR. Eigenvalue analysis is thus shown to be an effective technique for predicting the potential of SSR in wind farms connected to series compensated lines.

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