

DESIGN OF FIR AND IIR FILTERS

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Abstract—This paper based on the design of digital filter. Digital filters are basically of two types- FIR filter and IIR filter, digital IIR filter with fixed point representation is to be designed. Firstly digital low pass IIR filter is to be designed. Digital Filters are designed by using the values of both the past outputs and the present input, an operation brought about by convolution. If such a filter is subjected to an impulse then its output need not necessarily become zero. The impulse response of such a filter can be infinite in duration. Such a filter is called an Infinite Impulse Response filter or IIR filter. The infinite impulse response of such a filter implies the ability of the filter to have an infinite impulse response. This indicates that the system is prone to feedback and instability. In this paper, the design of IIR filters was considered. Several results from theory were verified in the design. The characteristics of a number of important approximations – Butterworth and Kaiser Window Filters were affirmed from the results obtained. The design of the low-pass filter was particularly insightful in comparing the relative merits and demerits of FIR and IIR filters in general as well as the individual IIR filter approximations.

Keywords—Infinite impulse response, filter design, linear phase filters FIR Filters, Butterworth filter, Kaiser Window filter

I. INTRODUCTION

The Digital Filter Design problem involves the determination of a set of filter coefficients to meet a set of design specifications. These specifications typically consist of the width of the passband and the corresponding gain, the width of the stopband(s) and the attenuation therein; the band edge frequencies (which give an indication of the transition band) and the peak ripple tolerable in the passband and stopband(s). Two types of digital filters exist – the IIR (Infinite Impulse Response) and the FIR (Finite Impulse Response). FIR filters were studied in an earlier paper. They possess certain properties, which make them the preferred design choices in numerous situations over IIR filters. Most notably, FIR filters (all zero system function) are always stable, with a realization existing for each FIR filter. Another feature exclusive to FIR filters is that of a linear phase response. The content of this paper is the design of IIR filters using MATLAB [1]. The design of IIR filters proceeds through a vastly different set of steps than those followed by FIR filter design algorithms. The design of IIR filters is closely related to the design of analog filters, which is a widely studied topic. An analog filter is usually designed and a transformation is carried out into the digital domain. Two transformations exist – the impulse invariant transformation and the bilinear transformation. In this paper, the focus is on designing minimum order IIR filters to meet a set of specifications using MATLAB functions. The bilinear transformation has been used. The designed IIR filters are characterized by significantly lower order than the corresponding FIR filters. This comparison is brought about in Section I of the design where FIR filters are designed along with the IIR filters. There it is shown that the best IIR filter is less complex than the optimum FIR filter. The price to pay is the non-linear phase of the IIR filters, which is unavoidable [2].

II. BUTTERWORTH FILTERS

Butterworth filters are causal in nature and of various orders, the lowest order being the best (shortest) in the time domain, and the higher orders being better in the frequency domain. Butterworth or maximally flat filters have a monotonic amplitude frequency response which is maximally flat at zero frequency response (Fig 3.1) and the amplitude frequency response decreases logarithmically with increasing frequency[3]. The butterworth filter has minimal phase shift over the filter's band pass when compared to other conventional filters

$$\overline{B(\omega)}B(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}}$$

III. KAISER WINDOW FILTERS

The width of the main lobe is inversely proportional to the length of the filter. The attenuation in the side lobe is, however, independent of the length and is function of the type of the window. A complete review of many window functions and their properties was presented by Harris [2]. Therefore the length of the filter must be increased considerably to reduce the main lobe width and to achieve the desired transition band. Kaiser has chosen a class of windows having properties closely approximating those of the prolate spheroidal wave functions [4]. This family of windows, known as the Kaiser windows is defined by

$$w_k(\beta, n) = \frac{I_0\left\{\beta\left[1 - \left(\frac{2n}{N-1}\right)^2\right]^{1/2}\right\}}{I_0(\beta)} \quad -\frac{N-1}{2} \leq |n| \leq \frac{N-1}{2}$$

otherwise

Where N is window length and $I_0(x)$ is the modified Bessel function of the first kind of order zero, given by:

$$I_0(x) = \sum_{k=0}^{\infty} \left[\frac{(x/2)^k}{k!} \right]^2$$

The Kaiser window provides the designer considerable flexibility in meeting the filter specifications.

IV. DESIGN OF IIR AND FIR FILTERS

The Signal Processing Toolbox in MATLAB includes several useful functions for designing both FIR and IIR digital filters as well as traditional analog filters. The design techniques for the digital filters were briefly described in the previous section. The basic characteristics of the common analog filter types were also summarized. The filters considered in this paper are the Butterworth, and Kaiser Window.

As mentioned earlier, IIR filter design proceeds by conversion of digital specifications to analog specifications and an analog filter is then designed prior to conversion to the required digital filter using the appropriate frequency transformation. In this problem all the analog filter types are considered, making the problem a useful example for comparison of different design options. Then the same specifications are applied to the design of an FIR filter. This too involves the window method using, a Kaiser Window. Ultimately, this design problem allows a close comparison of all filter design options. All the examples are accompanied by the corresponding frequency response characteristics (both magnitude and phase plots), with close-ups given wherever required, impulse response diagrams (truncated to a significant number of terms – in our examples 60 when

infinite), and pole-zero diagrams [5,6]. All the magnitude response plots are in dB and phase response plots in radians. The frequency axis is in terms of normalized frequency.

• **Design parameter of filters**

This section involves the design of a low-pass filter with specifications as follows:

Sampling Frequency, $F_s = 10$ Hz

Passband Edge Frequency, $F_{c1} = 2$ Hz

Stopband Edge Frequency, $F_{c2} = 2.4$ Hz

Passband Ripple 0.1 dB

Minimum Stopband Attenuation 30dB

The IIR filter Butterworth were designed to meet these specifications.

Then the design of an FIR filter Kaiser Window was considered.

The results of each design are specified and then they are compared.

(A) IIR filter (Butterworth Filter)

Order = 21

Frequency Response of the Butterworth Filter

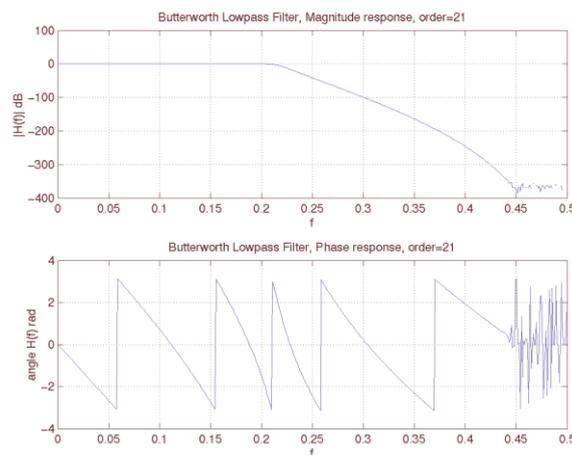


Fig. 1 Frequency response of Butterworth filter

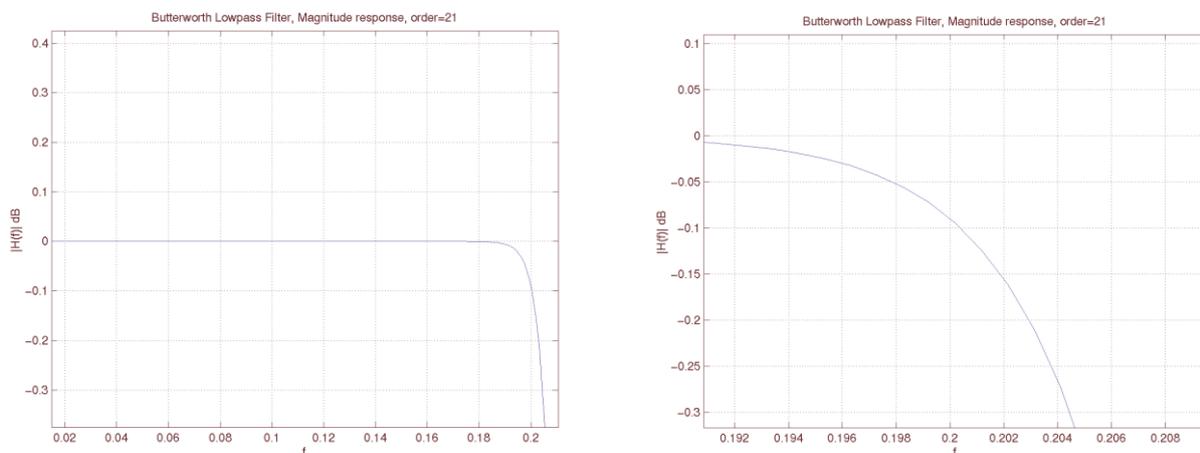


Fig. 2 Passband of Butterworth filter

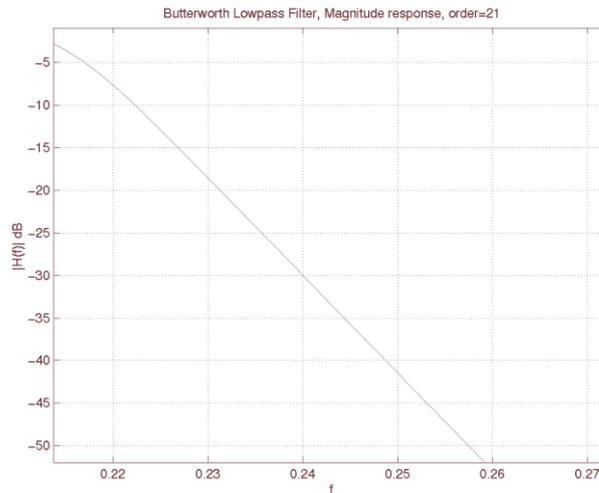


Fig. 3 Stopband of Butterworth filter

The following observations can be made about the frequency response of the Butterworth filter (Figs.1-3):

1. The designed filter meets the specifications.(Fig 2 and Fig 3)
2. The magnitude response is flat in the passband. This is called a maximally flat response.
3. The magnitude response is monotonic in the stopband.
4. The phase response is approximately linear in the passband,

Impulse Response and Pole Zero Diagram of the Butterworth Filter

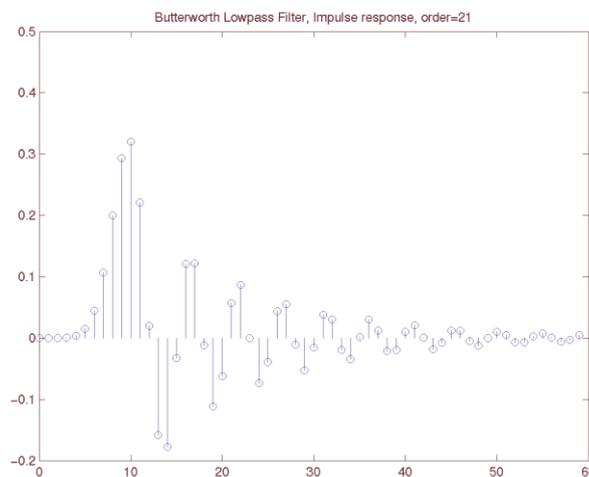


Fig. 4 Impulse response of Butterworth

filter from the impulse response (Fig-4), which decays with time, we can infer that the designed filter is stable. This is affirmed by the pole-zero diagram (Fig 5), in which all the poles reside within the unit circle. The pole zero diagram also contains 21 zeros corresponding to the roots of $(1+z^{-21})$ term that appears in the numerator of the transfer function on account of the bilinear transformation used by the butter function in MATLAB.

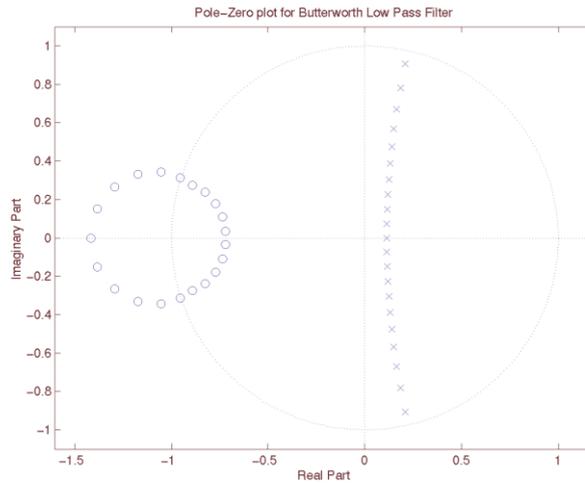


Fig. 5 Pole-Zero diagram for Butterworth Filter

(B) FIR Filters (Kaiser Window)

Order = 55

Frequency Response of the FIR Filter

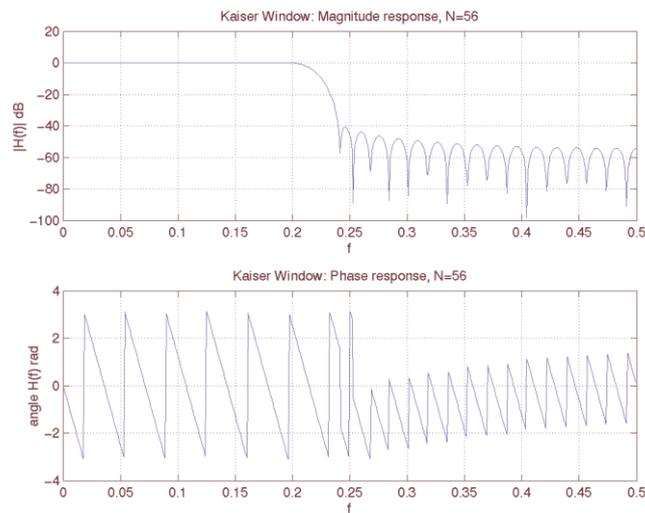


Fig. 6 Frequency response of FIR filter (using Kaiser Window)

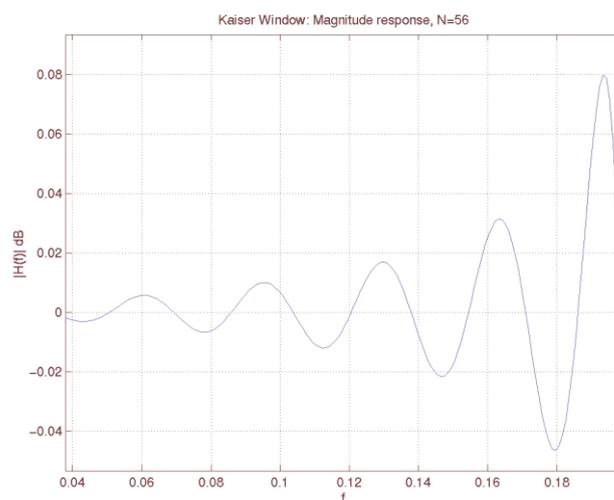


Fig. 7 Passband edge ripple

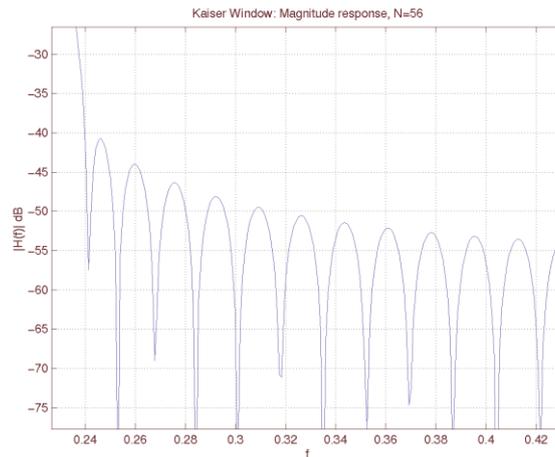


Fig. 8 Stopband of FIR filter designed using Kaiser window

The subject of FIR filter design using windows was studied in depth in an earlier project. The following characteristics can be observed from the frequency response (Figs 6-8):

1. The designed filter meets the specifications. (Fig. 7 and Fig. 8)
2. The passband contains ripples of increasing magnitude with the peak ripple appearing at the band-edge.
3. The stopband also consists of ripples of decreasing amplitude.
4. The ripples appear on account of the ripples in the frequency response of the Kaiser window [7]
5. The order of the FIR filter is 55, which is considerably higher than those obtained for any of the designed IIR filters. This is a general result.
6. The phase response is linear. This is a highly desirable property

Impulse Response and Pole Zero Diagram of the FIR Filter

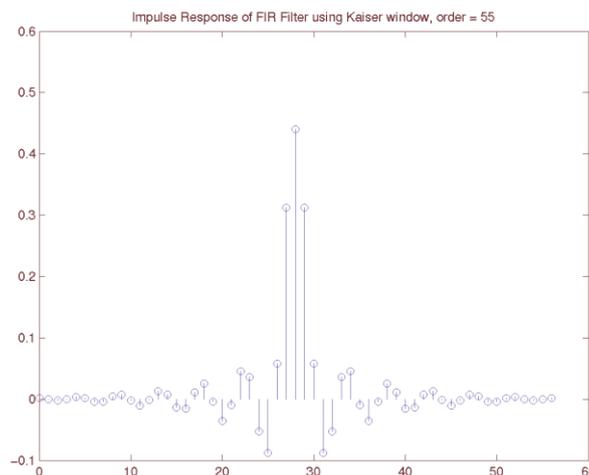


Fig. 9 Impulse response of FIR filter (Kaiser Window)

The impulse response Fig- 9 and pole-zero diagram Fig-10 exhibit some familiar properties the impulse response is symmetric about its midpoint. The filter has only zeros, and every zero is accompanied by the inverse of its conjugate. Consequently this filter will always be stable.

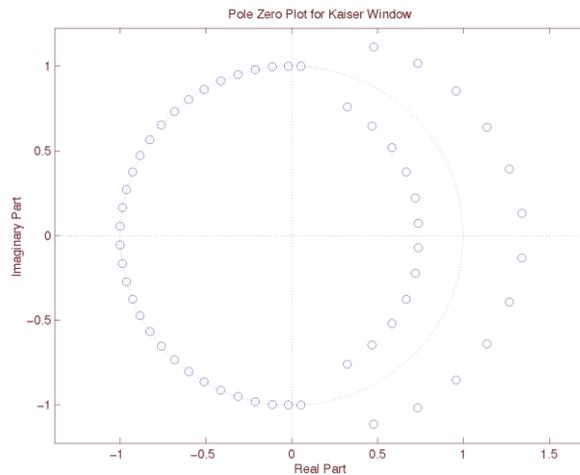


Fig.10 Pole-Zero diagram of FIR filter (Kaiser Window)

V. CONCLUSION

Comparison of the Designed Filters

The problem of digital low-pass filter designed was approached from all possible angles. This offers a means for comparison of the different options available to a designer. The results of the previous section are summarized below.

Type of Filter	Order
IIR Butterworth	21
FIR Kaiser Window	55

Table .1

All the designed filters meet a common specification. The obvious comparison is that between FIR and IIR filters. The IIR filters were found to have the following advantages:

- They require a significantly lower order for the same specifications
- This means that their hardware requirements should be less. This advantage is not so emphatic in practice. It is found that FIR filters having an order 4 times that of the corresponding IIR filter possess the same complexity [3]. This arises by calculating the number of multipliers required to implement the design. The importance of realization is not emphasized enough in this project, since only design is considered. Several references [8,9] treat this topic in detail. On the other hand the FIR filters possess the following desirable properties:

- They are always stable
- They can be designed to have linear phase
- In the case of the optimum filter the passband ripple can be controlled. For the Kaiser window this is not true. In fact, in the designed filter, a tighter passband tolerance (0.09 dB) had to be specified so that the filter met the requirements. The ultimate choice depends on the type of application and budget. If budget were a constraint, the Elliptic filter, having minimum complexity would be the obvious choice. If linear phase were a necessity then an FIR filter would have to be the design choice.

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