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Relationship Between Per Unit Reactance and Per Unit Inductance

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Abstract—Generally it is convenient to express the power system parameters in per unit system. It is a myth in power system literature that per unit reactance is equal to per unit inductance as $\omega_{pu} = 1$. This paper presents correct relationship between per unit reactance and per unit inductance and shows that both are not equal. In this paper correct modeling of R-L circuit and capacitor circuit is also presented.

Keywords— Electrical Power System, Per Unit System

I. INTRODUCTION

Generally Power system calculations are expressed in per unit (pu) system due to various advantages. Therefore resistance, inductance, inductive reactance are presented in pu values. In power system literature and books it is a myth that value of pu inductance is equal to pu inductive reactance and ω_{pu} =1. As X = ωL and $\omega_{pu} = 1$, therefore in pu system $X_{pu} = L_{pu}$ [1-2]. Only reference [3] has expressed correct relationship. It states that "With a base time of 1 second all time constants are expressed in seconds. A per-unit reactance is related to a per-unit inductance by $X_{pu} = \omega L_{pu}$ so that the normal relationship between inductance and reactance is maintained. The per-unit inductance is not equal to the per-unit reactance." This paper is in support of aforesaid lines of reference [3]. Simple circuits are analyzed and it is shown that in pu system $X_{pu} = 376.99L_{pu}$ for 60 Hz power systems. Value of 'ω' given in this paper all over is for 60 Hz power system.

II. PER UNIT SYSTEM OF A SIMPLE R-L CIRCUIT

A simple R-L circuit is shown in Fig. 1.

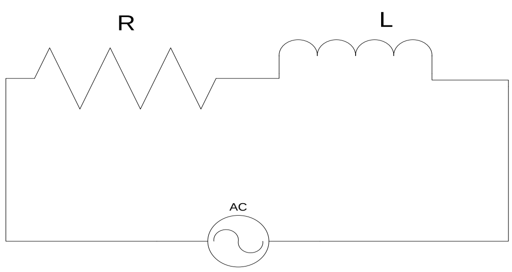

Figure 1. A simple R-L circuit

Its dynamic equations in d-q axis are as follows:

$$V_d = RI_d - \omega LI_q + L\frac{dI_d}{dt} \qquad (1)$$

$$V_q = RI_q + \omega LI_d + L\frac{dI_q}{dt} \qquad (2)$$

Here the units are: Voltage in volts, current in ampere, resistance in ohms, inductance in henry and time is in seconds. Steady state value of ‘ω’ is 376.99 rad/sec for 60 Hz system. I_d and I_q are the direct axis and quadrature axis components of current. Equations 1-2 can be transformed in per unit system by choosing base quantities. In pu system $time_{base}$ should be chosen ‘1’ second (i.e. $time_{base}$ = 1 second). Hence time in pu remains same and all the time constants remain same in pu as well as seconds. So, in transformation following relation must be maintained

$$\frac{R \text{ (in ohms)}}{L \text{ (in Henry)}} = \frac{R \text{ (in pu)}}{L \text{ (in pu)}} \quad \text{(i.e. Time contant in second and pu is same)} \qquad (3)$$

In transformation to all values in pu system, Let ‘R’ is reduced by a factor ‘k_1’ (in conversion from ohms to pu value) then to satisfy equation 3, ‘L’ must be reduced by factor ‘k_1’. To maintain ratio between ‘R’ and inductive reactance (X), in pu system in Fig. 1, ‘X’ also should be reduced by factor ‘k_1’. So, in conversion to pu system ‘X’ and ‘L’ both should be reduced by a factor ‘k_1’. It is possible when in pu system, relation between ‘X’ and ‘L’ remains ‘$X_{pu} = \omega L_{pu}$’ where ω = 376.99. (i.e. in pu system, ‘X’ and ‘L’ are not equal).

In the relation ‘$X_{pu} = \omega L_{pu}$’, a question may be asked, what should be the unit of ‘ω’. Since both ‘X’ and ‘L’ are in pu hence unit of ‘ω’ as rad/second cannot be acceptable. Therefore it is better to write unit of ‘ω’ as rad/pu.time in this relation (as time in seconds and pu are same).

Let in equation 1, ‘V_d’ reduced by a factor ‘k_2’ in conversion to pu system, then in this equation RI_d, LI_q and LdI_d must be reduced by a factor ‘k_2’. ‘ω’ remains same then only equation 1 gets satisfied. So in pu format, in equations 1-2, all quantities can be kept in pu but ‘ω’ remains equal to 376.99.

In IEEE First Benchmark model and various power system models, per unit inductive reactance ‘X’ is given instead of pu ‘L’, therefore to accommodate per unit ‘X’, equations 1-2 can be written as:

$$V_d = RI_d - \omega \frac{X}{\omega_0} I_q + \frac{X}{\omega_0}\frac{dI_d}{dt} \qquad (4)$$

$$V_q = RI_q + \omega \frac{X}{\omega_0} I_d + \frac{X}{\omega_0}\frac{dI_q}{dt} \qquad (5)$$

In equations 4-5, all the quantities are in pu. In steady state $\omega_0 = \omega = 376.99$. ‘X’ and ‘$\omega_0$’ are constant in transient period but ‘ω’ oscillates in transient period. As $\frac{\omega}{\omega_0} = 1$ therefore equations 4-5 can be reduced to

$$V_d = RI_d - \omega XI_q + \frac{X}{\omega_0}\frac{dI_d}{dt} \qquad (6)$$

$$V_q = RI_q + \omega XI_d + \frac{X}{\omega_0}\frac{dI_q}{dt} \qquad (7)$$

In equations 6-7 steady state value of ω is 1 pu which oscillates in transient period. ω_0 is equal to 376.99 which is constant in steady state period as well as transient period. If equations 1 and 6 are compared then misconception may be generated that pu values of inductive reactance and inductance is same which is not true. In equations 6 -7 ‘time’ is in pu therefore if transient response of any quantity (for example I_d) is drawn then I_d (on y- axis) and ‘time’ (on x-axis) both should be expressed in pu, but as $time_{base}$=1 second therefore in the literature ‘time’ axis is written in seconds instead of pu.

In equations 6-7, following quantities are variable in transient period: V_d, V_q, I_d, I_q, ω. While R, X, ω_0 are constant in steady state as well as in transient period. As 'X' is constant therefore it looks that in various test models (such as IEEE First Benchmark model), the value given in pu reactance is nothing but pu inductance as reactance and inductance are same in per unit format, but it is just a misconception. Though value of 'X' is fixed but in equations 6-7, oscillations of 'ω' represent variations in inductive reactance during transient period. Coefficients of $\dot{I}_d$ and $\dot{I}_q$ are fixed and represent inductances. (As X is pu reactance, ω_0 =376.99, Hence X/ω_0 represent pu inductance)

The linearisation of equations 6-7 is given here

$$\Delta V_d = R\Delta I_d - \omega_{0(pu)} X\Delta I_q - XI_{q0}\Delta\omega + \frac{X}{\omega_0}\Delta\dot{I}_d \qquad (8)$$

$$\Delta V_q = R\Delta I_q + \omega_{0(pu)} X\Delta I_d + XI_{d0}\Delta\omega + \frac{X}{\omega_0}\Delta\dot{I}_q \qquad (9)$$

Subscript '0' stands for initial conditions. Readers are requested to please note that $\omega_{0(pu)}$ and ω_0 are different. $\omega_{0(pu)}$ shows initial condition of 'ω' while ω_0 = 376.99 rad/sec. As $\omega_{0(pu)}$ = 1(Initial condition of ω is one in equations 6-7) , Hence equations 8-9 can be expressed as

$$\Delta V_d = R\Delta I_d - X\Delta I_q - XI_{q0}\Delta\omega + \frac{X}{\omega_0}\Delta\dot{I}_d \qquad (10)$$

$$\Delta V_q = R\Delta I_q + X\Delta I_d + XI_{d0}\Delta\omega + \frac{X}{\omega_0}\Delta\dot{I}_q \qquad (11)$$

III. PER UNIT SYSTEM OF VOLTAGE ACROSS CAPACITOR

Suppose voltage across a capacitor is $V_{cd} + jV_{cq}$ then d-q axis equations of voltage across the capacitor can be given as:

$$\dot{V}_{cd} = \omega V_{cq} + \frac{1}{C} I_d \qquad (12)$$

$$\dot{V}_{cq} = -\omega V_{cd} + \frac{1}{C} I_q \qquad (13)$$

Here the units are: Voltage in volts, current in ampere, capacitance in farads and time is in seconds. Steady state value of 'ω' is 376.99 rad/sec. and it oscillates in transient period.
In transformation to pu system $time_{base}$=1 second. In equations 12-13, Voltage, current and capacitance can be kept in pu, but value of 'ω' remains same i.e. 376.99, then only both the equations get satisfied.
If instead of capacitance, capacitive reactance is given in pu (generally in power system models per unit capacitive reactance is available) then for this purpose, equations 12-13 should be divided by 'ω_0'.

$$\frac{1}{\omega_0}\dot{V}_{cd} = \frac{\omega}{\omega_0} V_{cq} + \frac{1}{\omega_0 C} I_d \qquad (14)$$

$$\frac{1}{\omega_0}\dot{V}_{cq} = -\frac{\omega}{\omega_0} V_{cd} + \frac{1}{\omega_0 C} I_q \qquad (15)$$

Where $\omega = \omega_0$ = 376.99. 'ω' oscillates in transient period. 'ω_0' is constant in transient period. Now Equations 14-15 can be written as:

$$\frac{1}{\omega_0}\dot{V}_{cd} = \omega V_{cq} + X_C I_d \qquad (16)$$

$$\frac{1}{\omega_0}\dot{V}_{cq} = -\omega V_{cd} + X_C I_q \qquad (17)$$

Where, $\omega = 1$ pu, $\omega_0 = 376.99$.
In equations 16-17, following quantities are variable in transient period: V_{cd}, V_{cq}, I_d, I_q, ω. While X_c, ω_0 are constant in steady state as well as in transient period.
Linearisation of equation 16 is as follows:

$$\frac{1}{\omega_0}\Delta\dot{V}_{cd} = \omega_{0(pu)}\Delta V_{cq} + V_{cq0}\Delta\omega + X_C\Delta I_d \qquad (18)$$

$$\text{or, } \frac{1}{\omega_0}\Delta\dot{V}_{cd} = \Delta V_{cq} + V_{cq0}\Delta\omega + X_C\Delta I_d \qquad (19)$$

Similarly, linearisation of equation 17 is,

$$\frac{1}{\omega_0}\Delta\dot{V}_{cq} = -\Delta V_{cd} - V_{cd0}\Delta\omega + X_C\Delta I_q \qquad (20)$$

Lineaisation of non linear differential equations are necessary because eigenvalues of non linear differential equations cannot be obtained.

IV. CONCLUSION

There are lots of misconceptions in power system modeling. In this paper correct modeling of simple electrical systems in d-q axes and their linearization is presented and it is shown that in pu format inductive reactance cannot be equal to inductance. In power system literature it is mentioned that in per unit format inductive reactance is equal to inductance but in this paper it is proved that it is not correct.

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