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# A Multi-objective Zero-One Model for Determine the Optimal Portfolio Selection from a Set of Different Projects

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**Abstract**—A multi-objective binary integer programming model for R&D project portfolio selection with competing objectives is developed when problem coefficients in both objective functions and constraints are uncertain. Robust optimization is used in dealing with uncertainty in order to give the best solution from the model. An example is presented to illustrate the solution approach. The developed approach can be applied to general multi-objective mixed integer programming problems.

**Keywords**—multi-objective programming; robust optimization; imprecise information; Portfolio selection

## I. INTRODUCTION

Project is a short-term effort which is in order to produce products, provide services or obtain results.

A portfolio is a group of projects carried out under the sponsorship or management of a particular organization. Since there are not enough resources to carry out each and every project, these projects must compete against each other due to a constraint in the resources. (Manpower, finance, time and etc.)

Portfolio selection is a periodic activity which involves selecting the right project from the available projects and the projects entering the organization at every stage. To sum up, a portfolio helps to accomplish an organization's objectives without exceeding from the existing constraints or the limitation of the available resources.

## II. PROBLEM STATEMENT

The aim of the multi-objective R&D project portfolio selection problem is to select a subset as a portfolio from a large set of possible candidate projects considering multiple conflicting objectives, subjects to a set of constraints. Let K denote the number of objective functions, m the number of constraints, and n the number of candidate projects in the entire set. There is no prior requirement for the number of projects to be selected into the portfolio. Without loss of generality, all objective functions are assumed to be minimized. The multi-objective R&D project portfolio selection model is stated as in (1) in the following

$$\begin{aligned} &\text{Min } Z_k=f_k(x) \qquad \forall k \\ &\text{s.t. } g_i(x)\leq b_i \forall i \\ &x\in B^n. \end{aligned} \tag{1}$$

 

In this model, x is the vector of binary decision variables, $f_k(x)$ is the k th objective function, and $g_i(x) = \sum a_{ij}x_j \leq b_i$ is the i th constraint. Although each application is different, the objective function may include the maximization of total expected profit, maximization of expected market share, or minimizing the total expected risk, while the constraint may include limit budget, scarce human and material resources, and interdependence and interaction among the candidate projects

When a multi-objective programming is solved, many non- dominated solutions need to be generated as trial solutions. These non-dominated solutions are usually evaluated by the DM so as to elicit preference information from the DM. Non-dominated solutions are usually generated by solving augmented weighted Tchebycheff programs derived from the nominal model. The weighting vector space is defined as

$$W=\{w\in R^k \mid w_k > 0, \sum w_k = 1\} \qquad (2)$$

Any $w \in W$ is a weighting vector. For a given $w \in W$, an augmented weighted Tchebycheff program for the nominal model (1) is formulated as in (3) in the following

$$\begin{aligned} \min \quad & \alpha + \rho\sum(z_k - z_k^{**}) \\ \text{s.t.}\ & \alpha \geq w_k (z_k - z_k^{**}) \forall k \\ & z_k = f_k(x) \quad \forall k \\ & g_i(x) \leq b_i \forall i \\ & x_j \in \{0,1\} \forall j \\ & z_k \text{ unrestricted} \forall k \\ & \alpha \geq 0 \,. \end{aligned} \qquad (3)$$

where $\rho > 0$ is a small scalar. Usually $\rho = 0.001$ is sufficient.

Note that in the augmented weighted Tchebycheff program (3), each objective function is converted into a constraint and, hence, the number of objective functions is not a concern from a computational point of view.

## III. ROBUST OPTIMIZATION FOR R&D PROJECT PORTFOLIO SELECTION IN MULTI-OBJECTIVE PROBLEMS

Under uncertainty, the problem coefficient in (1) is uncertain and,hence, the selected portfolio must be robust, i.e., the solution should remain feasible (constrain robust), efficient and most preferred by the DM (objective function robust) under all possible realization of imprecise coefficient. The nominal value and the half-interval width of $c_{kj}$ are reported by $\bar{c}_{kj}$ and $\hat{c}_{kj}$ .The k th objective function is expressed as $\bar{f}_k(c_k , x) = \sum c_{kj}x_j$ where $c_k$ is the vector of imprecise coefficients in the k th objective function with each $\mathbf{c_{kj} \in [\bar{c}_{kj} - \hat{c}_{kj} , \bar{c}_{kj} + \hat{c}_{kj}]}$.

$\bar{f}_k (c_k , x)$ is a function of both $c_k$ and x, because each $c_{kj}$ is treated as a variable. The absolute value of the scaled deviation of $c_{kj}$ from its nominal value $\bar{c}_{kj}$, denoted by $\bar{\delta}_{kj}$, is defined in the following

$$\delta'_{kj} = \left| (c_{kj} - \bar{c}_{kj})/\hat{c}_{kj} \right| \forall\, k , j \,. \qquad (4)$$

A budget of uncertainty $\Gamma'_k$ is imposed to the k th objective function such that

$$\sum \delta'_{kj} \leq \Gamma'_k \quad j=1\ldots n \qquad 0 \leq \Gamma'_k \leq n, \qquad (5)$$

where $\Gamma'_k=0$ and $\Gamma'_k=n$ correspond to the nominal and worst cases.

Note that while $\Gamma_i$ controls the robustness of the i th constraint, $\Gamma'_k$ controls the robustness of the k th objective function against the level of conservatism. Imposing the budget of uncertainty for the constraints and the objective functions will ensure that the solution will remain both constraint robust and objective function robust. The non-linear robust formulation of the nominal model in (1) is stated as

$$
\begin{aligned}
\min \quad & z_k = \max\, [\bar{f}_k(c_k, x) \mid \sum \delta'_{kj} \le \Gamma'_k] \qquad \forall\, k \\
\text{s.t.} \max\, & [\bar{g}_i(a_i, x) \mid \sum \delta_{ij} \le \Gamma_i\,] \le b_i \; \forall\; i \\
& x \in B^n \\
& j = 1 \ldots n \\
& \bar{g}_i(a_i, x) = \sum a_{ij} x_j \quad , j = 1 \ldots n
\end{aligned}
\tag{6}
$$

each $c_{k\,j}$ in the objective function is considered imprecise in (6).

Any feasible solution to the above model is called a robust feasible solution.

$X^{\Gamma} = \{x \in B^n \mid \max\, [\bar{g}_i(a_i, x) \mid \sum \delta_{ij} \le \Gamma_i] \le b_i \;\; \forall i\}$ is called the robust feasible region in decision space for a given $\Gamma$ . A $x \in X^{\Gamma}$ is called a robust feasible solution in decision space. For given $\Gamma'$ and $\Gamma$, the set

$Z^{\Gamma,\Gamma'} = \{z \in R^k \mid z_k = \max\, [\bar{f}_k(c_k, x) \mid \sum \delta'_{kj} \le \Gamma'_k],\ x \in X^{\Gamma}\}$ is the robust feasible region in criterion vector. The robust ideal point $z^* \in R^k$ is defined as $z^*_k = \min \{\max\, [\bar{f}_k(c_k, x) \mid \sum \delta'_{kj} \le \Gamma'_k],\ x \in X^{\Gamma}\}$. A robust utopian point is also defined as $z^{**} \in R^k$ such that $z^{**}_k = z^*_k - \epsilon_k$ with $\epsilon_k > 0$ and small.

For a given weighting vector $w \in W$, a robust augmented weighted Tchebycheff program for the non-linear programming model in (6) is formulated from (3) as the following

$$
\begin{aligned}
\min \quad & \alpha + \rho \sum (z_k - z_k^{**}) \\
\text{s.t.}\ \alpha \ge\ & w_k\,(z_k - z_k^{**}) \qquad \forall k \\
& z_k = \max\, [\bar{f}_k(c_k, x) \mid \sum \delta'_{kj} \le \Gamma'_k] \quad \forall k \\
& \max\, [\bar{g}_i(a_i, x) \mid \sum \delta_{ij} \le \Gamma_i\,] \le b_i \forall i \\
& x_j \in \{0,1\} \forall j \\
& z_k \text{ unrestricted} \forall k \\
& \alpha \ge 0\,.
\end{aligned}
\tag{7}
$$

the coefficient in the objective function of model (7) are exactly known.

An optimal solution to (7) minimizes the augmented weighted Tchebycheff metric between $z^{**}$ and $z \in Z^{\Gamma,\Gamma'}$ while respecting the budget of uncertainty constraints. This solution to this formulation has some interesting features. First, it is a best solution for the selected $\Gamma$ and $\Gamma'$. Second, it is robust, i.e., insensitive to existing uncertainties in both the objective functions and constraints. These properties are significant because model (7) can assist the DM as a tool in finding best robust solution by balancing performance against robustness.

 

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## IV. AN ILLUSTRATIVE EXAMPLE

An IT company faces the selection of a portfolio from a total of n=12 projects where data on costs, benefits, and other related information for these projects are estimated. Existing cost interdependencies and synergic benefits among projects are also identified. There are m=2 resource constraint for hardware costs and software costs of the project that must be satisfied. The problem has K=3 objectives: maximization of total benefits, minimization of total risk scores, and minimization of total miscellaneous costs. The total hardware and software budgets are 30,000 and 7000. Table 1 and Table 2 present the problem data.

*Table 1 Original estimates for independent benefits, costs, and risk scores*

| Project | Mandated | Contingent | Annual benefit | Hardware costs | Software costs | Miscellaneous costs | Risk scores |
|---|---|---|---|---|---|---|---|
| 1 | Yes | - | 1500 | 15,000 | 2200 | 0 | 6 |
| 2 | No | 1 | 225 | 300 | 1500 | 0 | 5 |
| 3 | No | 2 | 110 | 250 | 250 | 0 | 2 |
| 4 | No | 2 | 110 | 400 | 400 | 0 | 2 |
| 5 | No | - | 2500 | 2400 | 2400 | 0 | 2 |
| 6 | No | - | 550 | 800 | 800 | 0 | 2 |
| 7 | No | 5 | 11 | 0 | 23 | 0 | 1 |
| 8 | No | 5 | 11 | 0 | 22 | 0 | 1 |
| 9 | No | 5 | 5 | 0 | 3 | 10,800 | 1 |
| 10 | No | 5 | 25 | 0 | 11 | 150 | 3 |
| 11 | No | 1 | 45,500 | 0 | 0 | 600 | 1 |
| 12 | No | 11 | 2000 | 0 | 0 | 2500 | 0 |

*Table 2 Original estimates for independent costs and benefits*

| Interdependent projects | Additional benefits | Shared hardware costs | Shared software costs |
|---|---|---|---|
| 2,3 | | | 255 |
| 2,4 | | | 125 |
| 3,4 | 65 | 164 | 250 |
| 4,5 | | 450 | 300 |
| 4,6 | | 350 | 275 |
| 5,6 | | 350 | 225 |
| 10,11,12 | 4500 | | |
| 4,5,6 | | 700 | 475 |

Similar to those in Santhanam and Kyparisis (1995), 20 binary variables ($x_j$) are defined and used to model the selection of a portfolio from n=12 projects as well as to model the 8 project interdependencies. The final linearized multi-objective binary integer programming model is formulated as (8) in the following

$$\min - f_1(x) = -1500x_1 - 225x_2 - 110x_3 - 110x_4 - 2500x_5 - 550x_6 - 11x_7 - 11x_8 - 5x_9 - 25x_{10} - 45{,}500x_{11} - 2000x_{12} - 65x_{3.4} - 4500x_{10.11.12}$$

 

$$\min f_2(x) = 6x_1 + 5x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + 3x_{10} + x_{11}$$

$$\min f_3(x) = 10{,}800x_9 + 150x_{10} + 600x_{11} + 2500x_{12}$$

$$\text{s.t.}\ \ 15{,}000x_1 + 300x_2 + 250x_3 + 400x_4 + 2400x_5 + 800x_6 + 164x_{3.4} + 450x_{4.5} + 350x_{4.6} + 350x_{5.6} + 700x_{4.5.6} \leq 30{,}000$$

$$2200x_1 + 1500x_2 + 250x_3 + 400x_4 + 2400x_5 + 800x_6 + 23x_7 + 22x_8 + 3x_9 + 11x_{10} + 255x_{2.3} + 125x_{2.4} + 250x_{3.4} + 300x_{4.5} + 275x_{4.6} + 225x_{5.6} + 475x_{4.5.6} \leq 7000$$

$$-x_1 + x_2 \leq 0,\ -x_2 + x_3 \leq 0,\ x_3 - x_4 \leq 0,\ -x_5 + x_7 \leq 0,\ -x_5 + x_8 \leq 0,\ -x_5 + x_9 \leq 0,$$

$$-x_5 + x_{10} \leq 0,\ -x_1 + x_{11} \leq 0,\ -x_{11} + x_{12} \leq 0,\ x_2 + x_3 - x_{2.3} \leq 1,\ -x_2 - x_3 + 2x_{2.3} \leq 2,$$

$$x_2 + x_4 - x_{2.4} \leq 1,\ -x_2 - x_4 + 2x_{2.4} \leq 0,\ x_3 + x_4 - x_{3.4} \leq 1,\ -x_3 - x_4 + 2x_{3.4} \leq 0,$$

$$x_4 + x_5 - (x_{4.5} + x_{4.5.6}) \leq 1,\ -x_4 - x_5 + 2(x_{4.5} + x_{4.5.6}) \leq 0,\ x_4 + x_6 - (x_{4.6} + x_{4.5.6}) \leq 1,$$

$$-x_4 - x_6 + 2(x_{4.6} + x_{4.5.6}) \leq 0,\ x_5 + x_6 - (x_{5.6} + x_{4.5.6}) \leq 1,\ -x_5 - x_6 + 2(x_{5.6} + x_{4.5.6}) \leq 0,$$

$$x_4 + x_5 + x_6 - x_{4.5.6} \leq 2,\ -x_4 - x_5 - x_6 + 3x_{4.5.6} \leq 0,\ x_{10} + x_{11} + x_{12} - x_{10.11.12} \leq 2,$$

$$-x_{10} - x_{11} - x_{12} + 3x_{10.11.12} \leq 0.$$

$$x_1 = 1$$

$$x_j \in \{0,1). \quad \forall j \tag{8}$$

For each imprecise coefficient in the objective function and in the constraints of the project portfolio selection model, a value is randomly selected from its uncertainty interval. These selected coefficient values, instead of nominal values, are then use in model (8) to formulate the project portfolio selection model. The formulated model is not solved but is used to evaluate the final solution of the nominal model and of the robust model. The best solution reported in Santhanam and Kyparisis (1995) and Ringuest and Graves (2000) are also evaluated with the formulated model. Each set of randomly selected coefficient values represent one realized instance of the imprecise coefficients in the project portfolio selection model. A total of 10,000 sets of randomly selected coefficient values are generated. Each solution is then evaluated by the constraints of the formulated model to determine if it is feasible. Only when the solution is feasible, it is evaluates with the three objective function of the formulated model to obtain the corresponding values for each $z_k$.

## V. CONCLUSION

The problem of selecting a portfolio of R&D projects is considered when there are multiple conflicting objectives and when there are uncertainties in problem data including objective function and constraint coefficients. The final portfolio is most preferred by the DM and is robust in terms of all possible realizations of imprecise problem coefficients.

A noticeable point of the proposed approach is that this robustness is achieved without bothering the DM in supplying unknown distribution details for the imprecise coefficients which is a major inconvenience in practical application. Also, this approach can be extended to other multi-objective mixed integer programming problems with uncertainties existing in both objective function and constraint coefficients**.**

 

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